A FINITE ELEMENT MODEL FOR THE STUDY OF THE CREEP AND SHRINKAGE EFFECTS IN THE PARTIALLY PRESTRESSED CONTINUOUS COMPOSITE BEAMS

Mazin Burhan
Asst. Lecturer , Civil Eng., Eng. College, Tikrit Univ.

ABSTRACT

A finite element model for the study of the creep, shrinkage and tensile stiffness between cracks effects in partially prestressed continuous composite beams with deformable shear connections is proposed in this paper.

The algorithm used is very general and allows the adoption of an arbitrary viscous law expressed in integral form and any load history. By means of simple numerical comparisons, the potentials of the proposed formulation are shown. This model, due to its simplicity, can be easy implemented in an analytical program.

KEYWORDS

Composite, creep, shrinkage, cracking, prestressed, continuous beams, flexible connection.

INTRODUCTION

Continuous composite beams, like all other structural members, must be designed to satisfy the requirements of both the strength and serviceability limit states. Design for adequate flexural strength is relatively straightforward, with plastification in sagging bending and buckling in negative bending generally governing the ultimate strength of an individual cross section [1].

The design of serviceability, however, is not as straightforward, since the prediction of the behavior under sustained service load is complicated by time-dependent deformation in the concrete due to creep and shrinkage and additional nonlinearity due to concrete cracking.

The most powerful way that can prevent concrete cracking and improve the stiffness and strength is using prestressed concrete.

Dezi and Tarantino [2] in 1993 and Dezi, et.al. [3] in 1995 studied the viscoelastic analysis of non-prestressed or fully prestressed composite continuous beams with flexible shear connector assuming that the concrete is uncracked.

Gilbert and Bradford [4] in 1995 described a simple analytical model of the behavior non-prestressed composite continuous beams under sustained service load using the age-adjusted effective modulus but with neglecting of the effects of the slip between the concrete slab and steel section as well as the tensile stiffness of concrete between cracks.

This paper describes dependent behavior of partially prestressed composite continuous beams by a numerical model which is constructed using finite element method, making it possible to evaluate the effects of concrete creep and shrinkage and stiffness of concrete between cracks on the deformations and stresses of the continuous composite beam at any time.

In this model, a numerical algorithm providing a result of good accuracy, has been obtained. The integration in time of viscous law of the material expressed in integral form (Bazant 1972)[5] is carried out by means of the trapezoidal rule.

Its validity is illustrated by simple comparison with experimental work done by Gilbert and Bradford in 1995.[4]

FINITE ELEMENT MODEL FORMULATION

The proposed model takes into consideration the composite beam of Fig.(1) constituted by two parallel beams, the first (concrete) with a viscous-elastic behavior with area A_c, moment of inertia J_c, and elastic modulus $E_c(t)$, the second (steel) of area A_s, moment of inertia J_s, and elastic modulus E_s.

One can suppose that the two beams are linked by means of a continuous system of elastic connectors of rigidity K(x) (which is sufficiently correct in serviceability condition) as in Fig. (1-c).

For the hypotheses which have been made, indicated by u_c,u_s the horizontal displacement of the point on the centroidal axis of the component beams, and v_c, v_s the vertical displacement of the same points, one obtains :

$$v_c = v_s = v \tag{1}$$

while the relative slip γ between steel and concrete .

$$\gamma_{(x)} = h_o \, [dv/dx] - (u_c - u_s) \tag{2}$$

Having indicated with S the slip force acting on the connection, one will obviously have:

$$S_{(x)} = K_{(x)} \, \gamma_{(x)} \tag{3}$$

CONSTITUTIVE EQUATION OF THE MATERIALS.

Generally, the concrete slab may contain three different materials; concrete, reinforcement , and prestressing steel. Because of the wide difference among the properties of these three materials, as well as difference between the behavior of concrete in tension and in compression, the layerd model approach will be useful for the analysis of such complex element (Fig.2).

The concrete layers under compression is assumed to be a linear visco-elastic material governed by well-known integral-type relationship between a prescribed uniaxial stress history and corresponding strain(Bazant 1972):

$$\varepsilon_c(t,t_o) = \int_{t_o}^{t} \Phi(t,\tau)\, d\sigma_c(\tau) + \varepsilon_n(t) \tag{4}$$

where ε (t , to) is the total strain at time to ; which is the sum of the visco-elastic strain developed in the time interval (t,to) under examination, represented by Stieltjes integral :

$$\varepsilon_c(t,to) = \int_{to}^{t} \Phi(t,\tau)\, d\sigma_c(\tau)$$

and the inelastic strain $\varepsilon n(t)$, which is independent of the stress state,due ingeneral to the effect of the shrinkage and the thermal dialation.

The function Φ(t,τ), called the viscosity function or creep function (usually provided by the codes) represents the total visco-elastic strain at time t caused by the application of a constant unitary stress applied at time τ. In the CEB code models [1976-1978], this function is defined as:

$$\varphi_{28}(t,\tau) = \frac{\varepsilon_{CV}(t,\tau)}{\sigma_{Co} / E_c 28} \tag{5}$$

The sum of an initial elastic contribution 1/Ec(τ) at time τ and of contibution of viscous character [$\varphi_{28}(t,\tau)/(Ec_{28})$ where Ec_{28} is the concrete modulus at 28 days and $\varphi_{28}(t,\tau)$ is the creep coefficient defined as the ratio between the strain of viscous character at time t {ε(t,τ)}, and the initial elastic strain for a stress $\sigma c_o = \sigma c(\tau)$:

$$\Phi(t,\tau) = \frac{1}{E_c(\tau)} + \frac{\varphi_{28}(t,\tau)}{E_c 28} \tag{6}$$

A very simple way[3] to overcome the difficulties induced by Voltarie's linear equation (4), consists in approximating the Stieltjes integral through the the application of trapezoidal rule[6].

For a law $\sigma_c(\tau)$ defined in the time interval (t_o,t_k) ; working thus one obtains:

$$\varepsilon_c(tk) = \sum_{i=1}^{k} \frac{1}{2}[\Phi(t_k,\tau_i) + \Phi(t_k,\tau_{i-1})]\Delta\sigma c_i + \varepsilon n(t_k) \tag{7}$$

with: $\Delta\sigma_{ci} = \sigma_c(t_i) - \sigma_c(t_{i-1})$

The constitutive relation in the form (7) is then easly utilizable by using an incremental step-by-step procedure. If one writes (7) for $t=t_{k-1}$ and subtract from (7) the equation obtained, we easly arrive at the relation:

$$\Delta\varepsilon_c(t_k) = \varepsilon_c(t_k - 1, to) - \varepsilon_c(t_{k-1}, t_o) = \frac{\Delta\sigma_{c_k}}{\overline{E}(t_k)} + \sum_{i=1}^{k-1} \frac{\Delta\sigma c_i}{2} R(t_k, t_i) + \Delta\varepsilon n_k \tag{8}$$

$$\overline{E}c(t_k) = \frac{2}{\Phi(t_k,t_k) + \Phi(t_k,t_{k-1})} \tag{9}$$

$$R(t_k,t_i) = [\Phi(t_k,t_i) + \Phi(t_k,t_{i-1}) - \Phi(t_{k-1},t_i) - \Phi(t_{k-1},t_{i-1})] \tag{10}$$

$$\Delta\varepsilon v_k = \Delta\varepsilon n(t_k) - \Delta\varepsilon n(t_{k-1}) \tag{11}$$

$$\Delta\varepsilon v_k = \sum_{i=1}^{k-1} \frac{\Delta\sigma c_i}{2} R(t_k, t_i) \tag{12}$$

It is therefore possible to porpose the constitutive equation of the concrete (8) in incremental form as:

$$\Delta \sigma c_k = \bar{E}c(t_k)\ [\Delta\ \varepsilon c_k - \Delta\ \varepsilon v_k - \Delta\ \varepsilon n_k] \quad (13)$$

where the increments of total $\Delta\ \varepsilon c_k$, viscous $\Delta\ \varepsilon v_k$ and inelastic $\Delta\ \varepsilon n_k$ strain clearly appear in interval 'k'.

The stiffness of concrete between cracks is considered in this research by adopting an incremental stress-strain relationship for the reinforcement and prestressing steel including this effect at the crack position.

When a crack is formed, the stress in the concrete immediately adjacent to the crack drops to zero and the steel stress increases to that corresponding to the fully cracked state. With increasing distance from the crack , the stress in the concrete increased as force is transferred from the steel to the concrete by bond stress, τ_b , until at some distance , ℓt , from the crack , the concrete again attains the tensile strength of the concrete, Fig. (3) .

The empirical formula suggested by CEB [1967-1978] for crack spacing , ℓ (in mm) , that will be recommended here is :

$$\ell = 2(c+s/10)+(k_2\ f_{ct}/\ \tau_b)\ (\ A_{ct}/u_s) \quad (14)$$

where c= the concrete cover (mm) ; s=spacing of bar (mm); A_{ct} = area of concrete in tension; u_s= the sum of the perimeters of the reinforcing bars; and k2 = a coefficient equal equal 1.0 for pure tension and 0.5 for pure bending.

It assumed that the bond stress is independent of the displacement of the steel bar relative to the outer surrounding concrete. So, the distance ℓ_t is calculated as follows:

$$\ell_t = AsEr(\varepsilon_{r2} - \varepsilon_{cr}) / (\tau_b u_s)\ \ell_t \qquad (15)$$

where $\varepsilon c_r = fct / \bar{E}c(t)$; εr_2 = the reinforcing strain at the crack position , and Er = reinforcement steel young's modulus. With regard to the stress of the reinforcement at the crack position three stages can be distinguished; Fig. (3).

The incremental form of the stress at the crack position $\Delta\ \sigma r_{2k}$- mean strain $\Delta\ \varepsilon r_m$ relationship can be written as:

$$\Delta\ \sigma r_{2k} = E_r\ [\Delta\ \varepsilon r_{mk} + \partial\ \varepsilon r_k] \qquad (16)$$

Where $\partial\varepsilon r_k = \Delta\varepsilon r(t_k) - \Delta\varepsilon r(t_{k-1})$ and $\Delta\varepsilon r(t_{k-1})$ is memorised through the calculation.

$\Delta\varepsilon r(t_k)=0$ for $\ell_{tk} \leq 0.0$; or $\Delta\ \varepsilon r\ (t_k)=(1-\ell t/L)[\ \varepsilon r\ _2 - \ \varepsilon c_r]$ for $0<\ell_{tk}<\ell_k/2$ or $\Delta\ \varepsilon r(t_k) = [\tau_b u_s\ L]/[4\ E_s\ A_s]$ for $\ell t_k \geq \ell_{k/2}$, where ℓ_{tk}, ℓ_k are the values of ℓ_t,ℓ in time t_k.

In the same way, the behavior of prestressing steel in concrete slab of composite continuous may be governed by Eq. (16). It should be noted that the value of $\partial\varepsilon p_k$ must be calculated with the bond stress between the prestressing steel and the surrounding concrete.

ELEMENT FORMULATION

With references to the parameter of nodal displacement of the finite element proposed in Fig. (4) , according to the usual technique for finite elements, it is possible to suppose :

$$\Delta u_c(x) = \underline{N}_c\,\Delta\underline{u}_c$$
$$\Delta u_s(x) = \underline{N}_s\,\Delta\underline{u}_s \qquad (17)$$
$$\Delta v(x) = \underline{N}_v\,\Delta\underline{v}$$

Where

$\Delta\underline{u}_c^T=\{\Delta u_{c1},\Delta u_{c2},\Delta u_{c3}\}$; $\underline{N}_c=\{N_{c1}(x),N_{c2}(x),N_{c3}(x)\}$, and analogously for the vectors $\Delta\underline{u}_c^T$ and $\Delta\underline{v}^T$ and for matrices of shape function $\underline{N}_s$, $\underline{N}_v$. The expression of the components of which is reported in appendix A.

The incremental mean strain $\Delta\varepsilon_{ck}(x)$ of the concrete layer (i) can be expressed as a function of incremental nodal displacements

$$\Delta\varepsilon c_k(x)=\underline{N}_c'\,\Delta\underline{u}_{ck} - y_i\,\underline{N}_v''\,\Delta\underline{v}_k \qquad (18)$$

where the denote $'$ and $''$ is the first and seconed derivative.

It is possible to apply the principle of virtual work to a generic instant t_k, considering the creep effects and the load applied in the time interval (t_{k-1},t_k) and to obtain , when working with one finite element:

$$\int_{Vc}\delta(\Delta\varepsilon c_k)\Delta\sigma c_k dVc + \int_{Vs}\delta(\varepsilon s_k)\Delta\sigma s_k dVs + \int_L \delta(\Delta\gamma_k)\Delta S_k dx = \int_L \delta(\Delta v_k)\Delta q_k dx \qquad (19)$$

where v_c , v_s represent the volume of concrete and steel of the generic element and Δq_k represents the possible increment of distributed load applied to the element of length L in the time interval considered.

Considering constitutive relation (13) together with the relations (2) and (3), supposed in incremental form, for the (17)and (19) one obtain :

$$\int_L \{ \int_{Ac} [\delta(\Delta \underline{u}c_k{}^T)\underline{N}'c^T - y_c \delta(\Delta \underline{v}_k{}^T)\underline{N}''v^T]\overline{E}c(t_k)[\underline{N}'c\Delta\underline{u}c_k - yc\underline{N}''v\Delta\underline{v}_k - \Delta\varepsilon v_k - \Delta\varepsilon n_k]dAc\} +$$
$$\int_L \{ \int_{As} [\delta(\Delta \underline{u}s_k{}^T)\underline{N}'s^T - y_s \delta(\Delta \underline{v}_k{}^T)\underline{N}''v^T]\overline{E}s[\underline{N}'s\Delta\underline{u}s_k - y_s\underline{N}''v\Delta\underline{v}_k]dAs\}dx +$$
$$B\ hr \int_L \{ \int_{Ac} [\delta(\Delta \underline{u}c_k{}^T)\underline{N}'c^T - y_r \delta(\Delta \underline{v}^T{}_k)\underline{N}''v^T]Er[\underline{N}'c\Delta uc_k - y_r\underline{N}''v\Delta\underline{v}_k + \partial\varepsilon r_k]dx +$$
$$B\ hp \int_L \{ \int_{Ac} [\delta(\Delta \underline{u}c_k{}^T)\underline{N}'c^T - y_r \delta(\Delta \underline{v}^T{}_k)\underline{N}''v^T]Ep[\underline{N}'c\Delta uc_k - y_p\underline{N}''v\Delta\underline{v}_k + \partial\varepsilon p_k]dx +$$
$$\int_L [-\delta(\Delta \underline{u}c_k{}^T)\underline{N}c^T + \delta(\Delta \underline{u}s_k{}^T)\underline{N}s^T + h_o\delta(\Delta \underline{v}_k{}^T)\underline{N}'v^T]K(x)[-\underline{N}c\Delta\underline{u}c_k + \underline{N}s\Delta\underline{u}s_k + h_o\underline{N}'v\Delta\underline{v}_k]dx$$
$$= \int_L [\delta(\Delta \underline{v}_k{}^T)\underline{N}v\Delta q_k dx.$$

(20)

Here, it is assumed that the inelastic strain in concrete section to be uniform:

$$\Delta\varepsilon n_k = \Delta\overline{\varepsilon}n_k \qquad (21)$$

i.e the shrinkage will made no bending moment present in section and the viscous strain $\Delta\varepsilon v_k$ divided int o two $\Delta\varepsilon v_k{}^N$ components, one produced by the effect of the normal force in section,$\Delta\varepsilon_k{}^M$ and produced by the bending moment:

$$\Delta\varepsilon v_k = \Delta\varepsilon v_k^{\,N} + \Delta\varepsilon v_k^{\,M} \qquad (22)$$

with $\Delta\varepsilon v_k^N$ constant on A_c and $\Delta\varepsilon v_k^M$ linear with y_c $\Delta\varepsilon v_k^M$ and equal to $y_c \Delta\bar{\varepsilon} v_k^{\,M}$

for which one obtain;

$$\int_{Ac} \Delta\varepsilon v_k \; dAc = \Delta\varepsilon v_k^{\,N} Ac \qquad \int_{Ac} y_c \Delta\varepsilon n_k dAc = 0$$

(23)

$$\int_{Ac} y_C \Delta\varepsilon v_k \; dAc = \int_{Ac} y_c \Delta\varepsilon v_k^{\,M} dAc = Jc\Delta\bar{\varepsilon} v_k^{\,M} Ac \qquad \int_{Ac} \Delta\varepsilon n_k \; dAc = \Delta\bar{\varepsilon} n_k Ac$$

(24)

As the equation of the virtual works (20) must be respected for every congruent $\delta(\Delta\underline{u}_{ck}^T)$, $\delta(\Delta\underline{u}_{sk}^T)$and $\delta(\Delta\underline{v}_k^T)$, one immediately obtain the system of solving equations;

$$[\int_L (\underline{N}'c^T \bar{E}c(t_k) Ac \underline{N}'c + \underline{N}c^T K(x) \underline{N}c) dx] \Delta\underline{u}c_k + [\int_L (-\underline{N}c^T K(x) \underline{N}s) dx] \Delta us_k +$$
$$[\int_L (-\underline{N}c^T K(x) h_o \underline{N}'v) dx] \Delta\underline{v}_k = \int_L \underline{N}'c^T \bar{E}c(t_k) Ac \Delta\varepsilon v_k^{\,N} dx + \int_L \underline{N}'c^T \bar{E}c(t_k) Ac \Delta\bar{\varepsilon} n_k^{\,N} dx$$
$$+ \int_L \underline{N}'c^T E_r B \; h_r \partial\varepsilon r_k dx + \int_L \underline{N}'c^T E_p B \; h_p \partial\varepsilon p_k dx$$
$$[\int_L (-\underline{N}s^T K(x) \underline{N}c) dx] \Delta uc_k + [\int_L (\underline{N}'s^T EsAs \underline{N}'s + \underline{N}s^T K(x) \underline{N}s dx] \Delta us_k$$
$$+ [\int_L \underline{N}s^T K(x) h_o \underline{N}' v dx] \Delta\underline{v}_k = 0$$

(25)

$$[\int_L (-\underline{N}'v^T K(x) h_o \underline{N}c) dx] \Delta uc_k + [\int_L (-\underline{N}'v K(x) h_o \underline{N}'s) dx] \Delta us_k + [\int_L (\underline{N}'v^T K(x) h_o^{\,2} \underline{N}'v +$$
$$(\underline{N}''v^T (\bar{E}c(t_k) Jc + EsJs) \underline{N}''v] \Delta\underline{v}_k = \int_L \underline{N}v^T \Delta q_k dx + \int_L \underline{N}''v^T \bar{E}c(t_k) Jc \Delta\bar{\varepsilon} v_k^{\,M} dx$$
$$+ \int_L \underline{N}''v^T E_r B \; h_r y_r \partial\varepsilon r_k dx + \int_L \underline{N}'c^T E_p B \; h_p y_r \partial\varepsilon p_k dx$$

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which with the meanings of the symbols as obvious can be proposed in the form:

$$\underline{K}_k u_c u_c \Delta\underline{u}c_k + \underline{K}_k u_c u_s \Delta\underline{u}s_k + \underline{K}_k u_c v\ \Delta\underline{v}_k = \Delta\underline{f}u_{c_k}{}^{v} + \Delta\underline{f}u_{c_k}{}^{n} + \Delta\underline{f}u_{c_k}{}^{r} + \Delta\underline{f}u_{c_k}{}^{p}$$

$$\underline{K}_k u_s u_c \Delta\underline{u}c_k + \underline{K}_k u_s u_s \Delta\underline{u}s_k + \underline{K}_k u_s v\ \Delta\underline{v}_k = 0$$

$$\underline{K}_k v u_c \Delta\underline{u}c_k + \underline{K}_k v_c u_s \Delta\underline{u}s_k + \underline{K}_k vv\ \Delta\underline{v}_k = \Delta\underline{f}_k{}^{q} + \Delta\underline{f}v_k{}^{v} + \Delta\underline{f}v_k{}^{r} + \Delta\underline{f}v_k{}^{p}$$

(26)

where the effects of the reinforcement and the prestress considered in this research on the nodal force vectors appear clearly in equation (26).

Submatrices $\underline{K}_k u_c u_c$, $\underline{K}_k u_c u_s$, etc are reported in appendix B.

One can rearrenge the equations and carry out a static condensation procedure of the equations of the internal nodal parameters uc_3 and us_3[6], to obtain a reduced system of the type:

$$\underline{K}^{e}{}_{k\,(8*8)}\ \Delta\underline{u}^{e}{}_{k(1*8)} = \Delta f^{e}{}_{k(1*8)} \qquad (27)$$

in which the matrices $\underline{K}^{e}{}_{k}$= condensed stiffness matrix of the element, and $\Delta\underline{u}_{ck}{}^{eT}$= { $\Delta\underline{u}_{c1k}$,$\Delta\underline{u}_{c2k}$, $\Delta\underline{v}_{1k}$,$\Delta\underline{v}_{2k}$,$\Delta\underline{u}_{c3k}$, $\Delta\underline{u}_{s1k}$,$\Delta\underline{u}_{s2k}$, $\Delta\underline{u}_{v3k}$,$\Delta\underline{u}_{v4k}\}^{e}$ as shown in Fig.(4).

The values of the load sub-vectore $\Delta\underline{f}^{n}{}_{uck}$ and $\Delta\underline{f}^{q}{}_{k}$ can be easily obtained by integrating the shape functions:

$$\Delta fu_{ck}{}^{q} = \int_L \underline{N}v^T \Delta q_x dx = \Delta q_x M_1 \qquad \text{where } M_1{}^T = \{L/2, L^2/12, L/2, -L^2/12\}$$

(28)

$$\Delta f_k{}^{n} = \overline{E}cAc \int_L \underline{N}'c^T \Delta\varepsilon n_x dx = \overline{E}cAc\Delta\varepsilon n_k M_2 \qquad M_2{}^T = \{-1,1,0\} \quad (29)$$

Also, in similar way, the load sub-vectors , $\Delta \underline{f}u_{ck}{}^{p}$,and $\Delta \underline{f}u_{ck}{}^{r}$ $\Delta \underline{f}v_{ck}{}^{r}$ $\Delta \underline{f}v_{ck}{}^{p}$
Can be written in the following form:

$$\Delta \underline{f}u_{ck}{}^{r} = E_r B\ h_r K^{*} Sur_k \qquad \Delta \underline{f}v_{ck}{}^{r} = E_r B\ h_r K^{**} Svr_k$$

$$\Delta \underline{f}u_{ck}{}^{p} = E_p B\ h_p K^{*} Sup_k \qquad \Delta \underline{f}v_{ck}{}^{p} = E_p B\ h_p K^{**} Svp_k \qquad (30)$$

here:

$$Sur_k{}^{T} = \{\partial\varepsilon r1_k, \partial\varepsilon r2_k, 1\} \qquad Svr_k{}^{T} = \{\partial\varepsilon r1_k, \partial\varepsilon r2_k, 1, 1\}$$

$$Sup_k{}^{T} = \{\partial\varepsilon p1_k, \partial\varepsilon p2_k, 1\} \qquad Svp_k{}^{T} = \{\partial\varepsilon p1_k, \partial\varepsilon p2_k, 1, 1\} \qquad (31)$$

For the load sub-vectors $\Delta \underline{f}^{v}{}_{uck}$ and $\Delta \underline{f}^{v}{}_{vk}$, going back to in the matrices in Eq. (25)

$$\Delta \underline{f}u_{ck}{}^{v} = \int_L \underline{\mathrm{N}}'\mathrm{c}^{\mathrm{T}}\,\overline{E}c(t_k) Ac \Delta\varepsilon v_k{}^{N} dx = \int_{Vc} \underline{\mathrm{N}}'\mathrm{c}^{\mathrm{T}}\,\overline{E}c(t_k) Ac \Delta\varepsilon_{v_k}{}^{N} dvc \quad (32)$$

For the relations (12),(13), (25), and (26), it can be proposed in the form:

$$\begin{aligned}
\Delta \underline{f}u_{ck}{}^{v} &= \frac{1}{2}\overline{E}c(t_k)\sum_{i=1}^{k-1} R(t_k,t_i)\int_{Vc}\underline{\mathrm{N}}'\mathrm{c}\,\overline{E}c(t_i)[\Delta\varepsilon c_i - \Delta\varepsilon v_i - \Delta\varepsilon n_i]dvc \\
&= \frac{1}{2}\overline{E}c(t_k)\sum_{i=1}^{k-1} R(t_k,t_i)\{[\int_L \underline{\mathrm{N}}'\mathrm{c}^{\mathrm{T}}\,\overline{E}c(t_i) Ac \underline{\mathrm{N}}'\mathrm{c}\ \ dx]\Delta\underline{\mathrm{u}}\mathrm{c}_i - \Delta \underline{f}u_{ci}{}^{v} - \Delta \underline{f}u_{ci}{}^{n}\} \\
&= \frac{1}{2}\overline{E}c(t_k)\sum_{i=1}^{k-1} R(t_k,t_i)\{\underline{K}_i{}^{*}u_c u_c \Delta\underline{\mathrm{u}}\mathrm{c}_i - \Delta \underline{f}u_{ci}{}^{v} - \Delta \underline{f}u_{ci}{}^{n}\} \\
&= \frac{1}{2}\overline{E}c(t_k)\sum_{i=1}^{k-1} R(t_k,t_i)\Delta \underline{r}u_{c_i}{}^{v} \qquad (33)
\end{aligned}$$

where $\Delta\underline{r}u_{ci}{}^{v}$ represents the increase in effective equivalent nodal force for the element due to creep effect in the i-th term interval. In order to facilitate the calculation of the term $\Delta\underline{f}u_{ck}{}^{v}$, it is then appropriate that the nodal force $\Delta\underline{r}u_{ci}{}^{v}$:

$$\Delta\underline{r}u_{c_i}{}^{v} = \underline{K}_i{}^{*}u_c u_c \Delta\underline{u}c_i - \Delta\underline{f}u_{ci}{}^{n} - \frac{1}{2}\overline{E}c(t_i)\sum_{j=1}^{k-1} R(t_i,t_j)\Delta\underline{r}u_{c_j}{}^{v} \qquad (34)$$

be memorised during calculation.

For the term $\Delta\underline{f}^{v}{}_{vk}$, in an analogous way the recursive relations are obtained;

$$\Delta\underline{f}v_k{}^{v} = \frac{1}{2}\overline{E}c(t_k)\sum_{i=1}^{k-1} R(t_k,t_i)\Delta\underline{r}v_k{}^{v} \qquad (35)$$

$$\Delta\underline{r}v_i{}^{v} = \underline{K}_i{}^{*}vv\Delta\underline{v}_i - \frac{1}{2}\overline{E}c(t_i)\sum_{j=1}^{k-1} R(t_i,t_j)\Delta\underline{r}v_j{}^{v} \qquad (36)$$

The matrices $\underline{K}_i^{*}u_cu_c$, $\underline{K}_i^{*}vv$ which appear in the equations (35), (36) are directly determinable from the matrices $\underline{K}_iu_cu_c$, $\underline{K}_ivv$ ignoring the terms in which the stiffness contribution due to the connection and steel bar appears.

Once $\Delta\underline{u}_k$ is obtained, it is then immediate to go back, for sake of congruence, to the nodal displacements of the element $\Delta\underline{u}^{e}_{ck}$ and consequently the increase in the characteristics of the stress and strain in the component beams.

The determination of the internal forces in the concrete must obviously be based on the constitutive equation (13). With the notation in Fig.(1) and omitting the symbol regarding the element, this leads to:

$$\Delta Nc_k(x) = \int_{Ac} -\Delta\sigma c_k \; dAc = \int_{Ac} -\overline{E}c(t_k)(\Delta\varepsilon c_k - \Delta\varepsilon v_k - \Delta\varepsilon n_k)dAc \tag{37}$$

$$\Delta Mc_k(x) = \int_{Ac} y_c \; \Delta\sigma c_k \; dAc = \int_{Ac} y_c \overline{E}c(t_k)(\Delta\varepsilon c_k - \Delta\varepsilon v_k - \Delta\varepsilon n_k)dAc \tag{38}$$

For the stress increment in the steel beam and the connection, as the elastic bond is linear, the usual technique of finite element is used.

EXPERIMENTAL VERIFICATION

To verify the applicability model , the comparison is made with the experimental work of Gilbert and Bradford (1992)[4] . they tested several two span non-prestressed continuous composite beams over a period of 340 days. The measured short-term and long-term deflections of two beams are compared here with the corresponding computed values. The two beams were continuous over two 5.8 m span and were identical, except for the load level, with cross section and elevation similar to that shown in Fig. (4). The relevant properties and are: D_c=70 mm ; b=1000 mm ; Ds=203 mm ; Ass=3230 mm^2; Iss = 23.6 * 10^6 mm^4 ; dss = 171.5 mm ; d_{sr} = 15 mm ; A_{sr} = 113mm^2 ; and L=5800 mm ; f_c =27 MPa.

The first beam B1 was subjected only to it's own weight, i.e. w=1.92kn/m, while the second beam B2 carried an additional superimposed sustained load of 4.75 kN/m, i.e. w =1.92+4.75=6.67 kN/m.

Measured and computed deflections for both beams are shown in Fig. (5) and (6), and are in reasonable agreement. The computer model appears to provide a reliable simulation of structural behavior.

In Fig. (7), the computed initial and final bending moment diagrams are plotted for beam B2. Also, shown is the final bending moment when shrinkage is set to zero. It can be seen that the relatively large time-dependent redistribution of moments that occur in continuous composite members at service load is primarily due to concrete shrinkage.

CONCLUSION

A computer model for calculating the time-dependent behavior of partially prestressed continuous composite beams with flexible shear connectors have been described. The analysis takes into account the material non-linearity caused by cracking of concrete slab in negative moment region and the stiffness of concrete between cracks and the time-dependent deformations caused by creep and shrinkage in the concrete. Computed results were shown to be in close agreement with laboratory measurements taken on two full-scale continuous composite beams (non-prestressed) for a period of 340 days. Results are also presented that demonstrate

APPENDIX A

Shape functions:

$$Nc_1(x) = Ns_1(x) = 1 - 3(\frac{x}{L}) + 2(\frac{x}{L})^2$$

$$Nc_2(x) = Ns_2(x) = -(\frac{x}{L}) + 2(\frac{x}{L})^2$$

$$Nc_3(x) = Ns_3(x) = 4(\frac{x}{L}) - 4(\frac{x}{L})^2$$

$$Nv_1(x) = 1 - 3(\frac{x}{L})^2 + 2(\frac{x}{L})^3 \qquad Nv_2(x) = x - 2(\frac{x^2}{L}) + 2(\frac{x^3}{L^2})$$

$$Nv_3(x) = 3(\frac{x}{L})^2 - 2(\frac{x}{L})^3 \qquad Nv_4(x) = -(\frac{x^2}{L}) + 2(\frac{x^3}{L^2})$$

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(94-100)

APPENDIX B

Stiffness sub-matrices of element with

$$\underline{K}u_c u_c = \begin{bmatrix} \frac{7}{3L}\overline{E}cAc + \frac{2}{15}KL & \frac{1}{3L}\overline{E}cAc - \frac{1}{30}KL & \frac{-8}{3L}\overline{E}cAc + \frac{1}{15}KL \\ & \frac{7}{3L}\overline{E}cAc + \frac{2}{15}KL & \frac{-8}{3L}\overline{E}cAc + \frac{1}{15}KL \\ Symm. & & \frac{16}{3L}\overline{E}cAc + \frac{8}{15}KL \end{bmatrix}$$

$$\underline{K}u_c u_s = \begin{bmatrix} -\frac{2}{15}KL & \frac{1}{30}KL & -\frac{1}{15}KL \\ & -\frac{2}{15}KL & -\frac{1}{15}KL \\ Symm. & & -\frac{8}{15}KL \end{bmatrix} = \underline{K}u_s u_c$$

$$\underline{K}u_s u_s = \begin{bmatrix} \frac{7}{3L}EsAs + \frac{2}{15}KL & \frac{1}{3L}EsAs - \frac{1}{30}KL & \frac{-8}{3L}EsAs + \frac{1}{15}KL \\ & \frac{7}{3L}EsAs + \frac{2}{15}KL & \frac{-8}{3L}EsAs + \frac{1}{15}KL \\ Symm. & & \frac{16}{3L}EsAs + \frac{8}{15}KL \end{bmatrix}$$

$$\underline{K}u_c v = \begin{bmatrix} \frac{1}{10}KH & -\frac{7}{60}KHL & -\frac{1}{10}KH & \frac{1}{20}KHL \\ \frac{1}{10}KH & \frac{1}{20}KHL & -\frac{1}{10}KH & -\frac{7}{60}KHL \\ \frac{4}{5}KH & \frac{1}{15}KHL & -\frac{4}{5}KH & \frac{1}{15}KHL \end{bmatrix} = \underline{K}u_s v = \underline{K}vu_c^{\,T} = \underline{K}vu_s^{\,T}$$

with :

$$z = \overline{E}cJcEsJs$$

$$\Delta f^{q^T} = [\frac{\Delta qL}{2}, \frac{\Delta qL^2}{12}, \frac{\Delta qL}{2}, -\frac{\Delta qL^2}{12}]$$

$$\underline{K}vv = \begin{bmatrix} \frac{12}{L^3}z + \frac{6}{5}KH^2 & \frac{6}{L^2}z + \frac{1}{10}KH^2 & -\frac{12}{L^3}z - \frac{6}{5}KH^2 & \frac{6}{L^2}z + \frac{1}{10}KH^2 \\ & \frac{4}{L}z + \frac{2}{15}KH^2L & -\frac{6}{L^2}z - \frac{1}{10}KH^2 & \frac{2}{L}z - \frac{1}{30}KH^2L \\ & & \frac{12}{L^3}z + \frac{6}{5}KH^2 & -\frac{6}{L^2}z - \frac{1}{10}KH^2 \\ Symm. & & & \frac{4}{L}z + \frac{2}{15}KH^2L \end{bmatrix}$$

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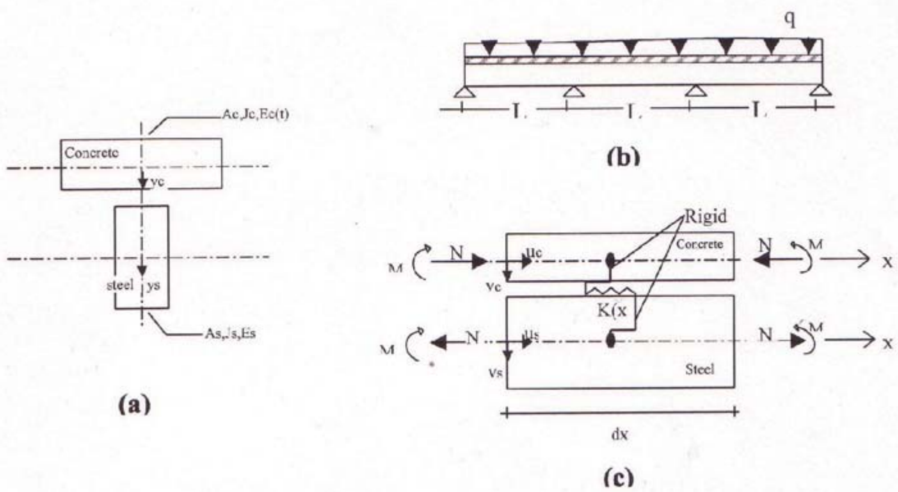

Fig.(1) *(a) Continuous composite beams; (b) cross section; (c) an elements connected only at nodes.*

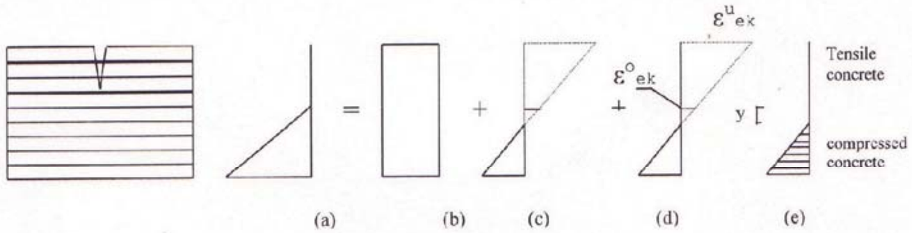

Fig.2 *General layered concrete slab: (a) mean total strain; (b) shrinkage strain; (c) creep strain; (e) stress distribution*

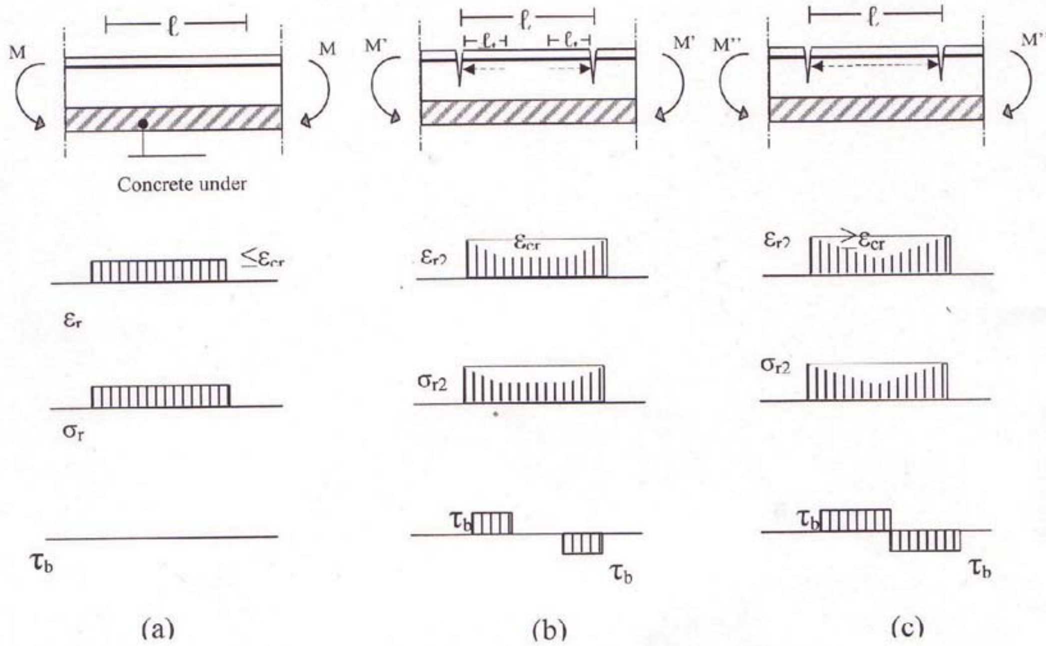

Fig.3 *The different stages of the strain and stress distribution between the cracks for the reinforcement in partially cracked slab. [M < M' < M'']*

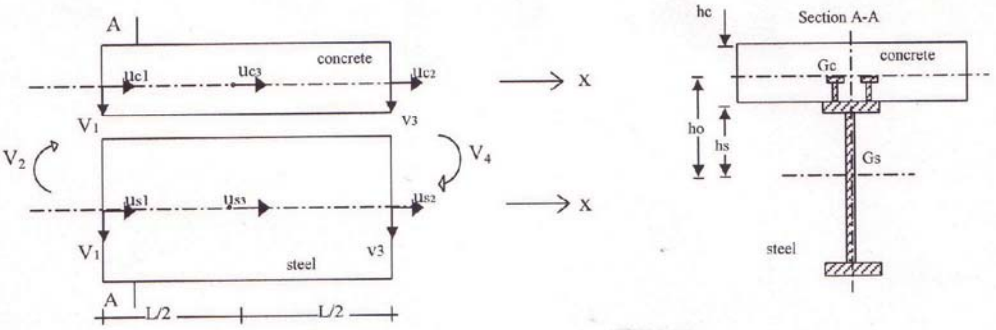

Fig. 4 *Proposed Finite element with nodal displacements and the cross section*

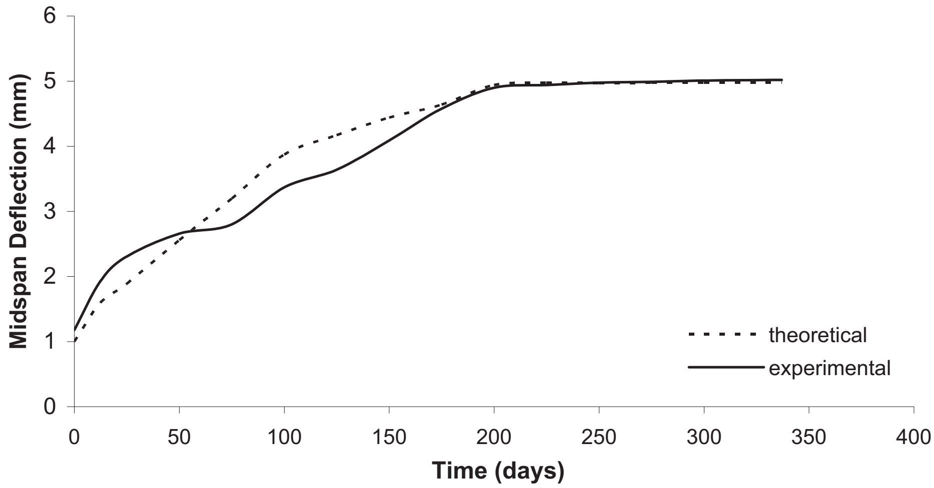

Fig.5 Mid-span Deflection versus Time Curves (Beam 1)

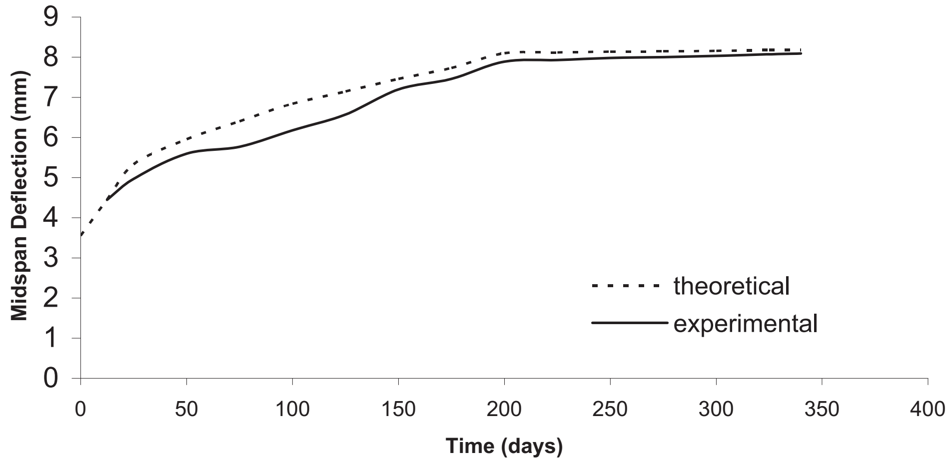

Fig.6 Mid-span Deflection versus Time Curves (Beam 2)

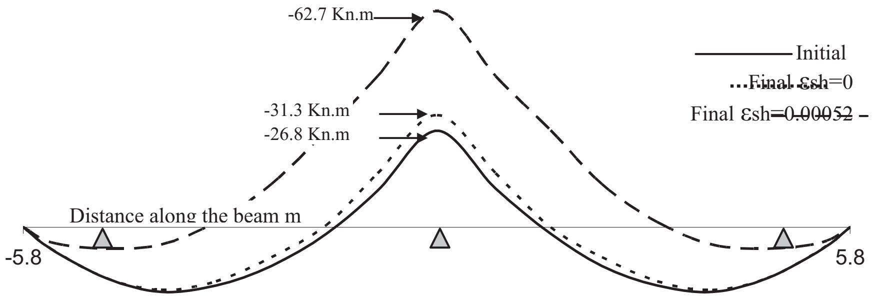

Fig.7 Initial and Final Bending Moment Diagrams for Beam2

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(100-100)

نموذج رياضي مبسط لحساب تأثيرات الزحف والانكماش في على العتبات المركبة المستمرة ذات الإجهاد المسبق.

مازن برهان الدين عبد الرحمن

مدرس مساعد، قسم الهندسة المدنية، كلية الهندسة، جامعة تكريت

الخلاصة

نموذج رياضي لحساب تأثيرات الزحف والانكماش في المنشآت المركبة المسبقة الإجهاد والتي تحتوي على روابط القص قد تم تطويره خلال هذا البحث.

تم إدخال تأثير الشقوق في الكونكريت بالإضافة إلى تأثير تصلب الشد بين الشقوق عن طريق تمثيلها بنماذج رياضية.

تم استخدام التقنية العددية (خطوة-خطوة) لحل صيغة التكامل الذي يصف علاقة الإجهاد-الانفعال للكونكريت، ونظراً لطبيعة المسألة الغير خطية، فقد تم حساب صلادة العنصر بالاعتماد على التقنية التكرارية لكل زيادة في الزمن وباستخدام طريقة نموذج الطبقات لتمثيل أجزاء المنشأ، هذا النموذج، بسبب بساطته، يمكن بسهوله تنفيذه في البرامج التحليلية.

النتائج الخاصة بهذا النموذج تم مقارنتها بالنتائج المختبرية لأحد المقاطع التي تم فحصها مسبقاً وقد كانت النتائج مطابقة بشكل عام.

الكلمات الدالة

العتبات المركبة، الزحف، الانكماش، التشقق، الإجهاد المسبق، العتبات المستمرة، روابط القص.