

STRESSES IN CONCRETE PAVEMENT SLABS ON REISSNER'S FOUNDATION SUBJECTED TO FLOATING TANDEM AXLES

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ABSTRACT

Presented in this paper are stress and load factor charts for floating tandem axles acting on the interior of concrete pavement slabs resting on Reissner's foundation. The floating tandem axle consists of dueted single axle followed by a single tired single axle or vice versa. The geometrical characteristics of the floating tandem axle were obtained from an axle load survey in Iraq. Based on the survey results, the floating tandem axle load frequency distribution histogram together with that for tire pressures were obtained. Circular tire-pavement contact areas between the tires of the floating tandem axle and the concrete pavement slab were assumed. The chart for the bending moment in concrete pavement slabs on Reissner's foundation developed by Reddy and Pranesh^[1] was used to determine the maximum bending moment due to interior loading by a floating tandem axle. The stress and load factor charts were developed for various slab thicknesses of 6, 8, 10, and 12 in. (15.24, 20.32, 25.40, and 30.48 cm). Using the developed stress charts, the maximum

bending stress in the concrete pavement slab can be obtained directly for given floating tandem axle load and slab thickness.

KEYWORDS

Concrete Pavement, Floating Tandem Axles, Load, Factor Charts; Reissner's Foundation; Stress Charts.

NOTATIONS

The following symbols are used in this paper:

A = contact area of one tire.

a = radius of circular contact area.

d = dual tire spacing.

E = modulus of elasticity of concrete.

E_f = modulus of elasticity of soil.

ESAL= equivalent single axle load.

ESWL= equivalent single wheel load.

FTAL= floating tandem axle load.

G = shear modulus of soil.

H = Depth of Reissner's foundation.

h = uniform thickness of pavement slab.

i = number (i=1,2,3).

K_a = axial modulus.

L_R = radius of relative stiffness of concrete pavement slab.

L_f = dimensionless load factor.

M = bending moment.

$M_{max.}$ = maximum bending moment in the interior of concrete

- pavement slab.
- n = direction.
- N_i = number of blocks enclosed within the i^{th} tire-pavement contact area on Reddy and Pranesh chart.
- N = total number of blocks enclosed on Reddy and Pranesh chart due to all tires of the three-wheel assembly.
- N_e = Number of blocks enclosed within contact area of the equivalent single wheel yielding maximum bending moment.
- O = origin of Reddy and Pranesh chart.
- P_e, P_f = tire pressure for the equivalent single wheel load and floating tandem axle respectively.
- S = axle spacing.
- μ = Poisson's ratio for concrete.
- α = Angle of rotation of floating tandem axle subtended between n -direction and direction of travel.

INTRODUCTION

A Portland cement concrete pavement or a rigid pavement consists generally of a Portland cement concrete slab resting either directly on the subgrade or on a subbase layer^[2]. The load carrying capacity of such a pavement is accomplished by the bridge action of the slab because the modulus of elasticity of concrete is much higher than that of the foundation material[3]. Therefore, variation in subgrade soil strength has little

significance upon the structural capacity of the pavement. The use of the subbase layer under rigid highway pavements is mainly for the control of pumping and not to improve the pavement structural capacity^[2]. Westergaard's^[4] work was the first serious theoretical attempt for rigid pavement design. In his solutions presented in 1927, he assumed that the subgrade to be a Winkler foundation and the slab of uniform thickness to be infinite in extent in all directions away from the load. Regarding the load, he assumed that for the case of interior loading of the slab, the load is distributed uniformly over a circular contact area. Pickett and Ray^[5] extended the work of Westergaard^[4] to include the effect of any loading configuration on bending moment and deflection of rigid pavement subjected to any multiple wheel loads. In fact, there are two methods for design of concrete roads. The first is based upon observations of the performance of full-scale roads such as that AASHTO^[6] design approach. The second one is based upon stress calculated in the pavement and the flexural strength of concrete^[1]. The soil media under rigid pavement is rather complex. Different models were introduced to represent the soil such as the Winkler model, Filonenko - Borodich model and Reissner model^[7]. The first model was covered by Razouki^[8] in determining the equivalent single axle load for floating tandem axle load on the interior of concrete pavement slab resting on Winkler foundation. The second model was covered by Razouki and Mohee^[9] in

developing stress charts for floating tandem axles on the interior of concrete pavement slabs on Filonenko - Borodich foundation. This Filonenko - Borodich model is presented by a modified Winkler model and has received recently great attention by AL – Lami^[10], AL – Muhanna^[11], and AL – Wazni^[12]. They pointed out that the maximum bending moment in the slab on Filonenko - Borodich foundation was less than that obtained using the Winkler foundation model. The majority of floating tandem axles studies were devoted to Winkler foundation^[13], Razouki and Hussain^[14,15]. The third model which is the Reissner model is based on continuum approach with the assumptions that in plane stresses throughout the continuum are negligibly small [i.e. $\sigma_x = \sigma_y = \tau_{xy} = 0$, in which σ_x & $\sigma_y =$ stresses in horizontal X & Y directions and $\tau_{xy} =$ shear stress in xy plane], and that the horizontal displacements at the upper and lower surfaces of the foundation layer are zero. This model as shown in Fig.1 is more general than Winkler model and retains the mathematical simplicity of Winkler model.

FLOATING TANDEM AXLES

As shown in Fig. 2, the floating tandem axle consists of two subaxles. The first subaxle is known as the leading axle and it consists of a single axle with dual tires on each end. The second subaxle consists of a single axle with a single tire on each end. A thorough study of characteristics of floating tandem axles in Iraq was carried out for the first time by Razouki and

Hussain^[14,15]. Another thorough study in Iraq was carried out by Mustafa^[16] and AL-Samarrai^[17]. These studies revealed that the predominating axle spacing (S, distance between the centers of the first and the second axles) was 130cm for 133 observations from out of 194 axles surveyed. Regarding the dual tire spacing (d, distance between centers of the dual tires of the first axle) the predominating value was 30cm for 140 observations out of 194^[16]. Thus the ratio $S/d = 130/30 = 4.33$ will be adopted in this work. From the same survey, the maximum floating tandem axle load was 26.13 tones (256.33kN) for loaded vehicles and the minimum floating tandem axle load was 5.60 tones (54.94kN).

STRESS CHARTS PARAMETERS

In order to develop design charts for floating tandem axles acting on the interior of concrete pavements on Reissner's foundation, it is desired to introduce dimensionless parameters to take into account the effect of various factors.

As discussed above, a good average value of 4.33 can be accepted for S/d ratio for developing the chart. The ratio that correlates the geometry of the axle with the geometry of the slab is d/L_R where L_R (radius of the relative stiffness of the concrete pavement) can be calculated from the equation (Reddy and Pranesh^[1]):

$$L_R = [Eh^3(CH)^2 / 12k_a(1 - \mu^2)]^{1/6} \quad \dots(1)$$

Where

L_R = radius of relative stiffness (m).

h = uniform thickness of concrete slab (m).

E = modulus of elasticity of the concrete slab (N/m^2).

$K_a = E_f / H$ = axial modulus (N/m^3).

E_f = modulus of elasticity of soil (N/m^2).

H = depth of foundation (m).

$C = (G_1 / 3E_f)^{.5}$, G_1 = shear modulus of soil (N/m^2).

μ = Poisson's ratio.

Thus, the effect of the properties of both the subgrade soil (represented by the axial modulus) and the concrete (represented by the modulus of elasticity and Poisson's ratio) are considered. There are two common shapes of contact area in use. The shape of first tire pavement contact area is circular while the second is consisting of a central rectangular portion with two semi-circles at the two ends of the rectangle in the longitudinal direction (direction of traffic movement). According to Razouki^[8], the effect of the shape of the contact area on the maximum bending moment on the interior of the slab is completely insignificant. Thus, the circular contact area will be adopted throughout this work for easement of calculations. Regarding the distribution of the floating tandem axle load upon the corresponding tires, the uniform distribution suggested by Razouki^[8] and Razouki and Hussain^[15] was accepted for the purpose of this work. For concrete pavement, a Poisson's ratio of 0.15 was accepted

throughout this work as this value was adopted by Reddy and Pranesh^[1] for developing their influence chart for determining the maximum bending moment on the interior of concrete pavement slab resting on Reissner foundation. Regarding the tire pressure of floating tandem axle, Al-Samarrai^[17] reported that the minimum tire pressure observed during his survey in Baghdad was 70psi (483.35kN/m²) and the maximum was 140psi (966.70kN/m²) with an average value of 107.66psi (743.39kN/m²). The Highway Design Manual^[18] reported that the maximum tire pressure allowed in Iraq is 95psi (655.975kN/m²). Therefore, the tire pressure of 95psi (655.975kN/m²) was accepted in order not to exceed the maximum allowable pressure in Iraq. In addition this maximum tire pressure is close to the average one of 107.66psi (743.39kN/m²) obtained from Al-Samarrai^[17] survey.

MAXIMUM BENDING MOMENT

In order to develop the design charts, the maximum bending moment due to floating tandem axle load should be determined first. The effect of one side of the floating tandem axle on the other side will be neglected^[16]. The bending moment can be calculated using the following equation (Reddy and Pranesh^[1]):

$$M_n = qL_R^2 N / 10000 \quad \dots(2)$$

where

M_n = bending moment at the origin in the n-direction
due to three-wheel assembly (N.m).

q = contact pressure = tire pressure for floating
tandem axle (N/m^2).

L_R = radius of relative stiffness (m.).

N = number of net blocks (positive blocks minus
negative blocks) enclosed by the contact areas.

To obtain the maximum bending moment due to the three-wheel assembly of one side of the floating tandem axle, the worst position and orientation of the assembly should be determined first. To do this, the effect of the lateral shift of the three-wheel assembly in the n-direction on the bending moment at the computational point (origin at O, see Fig.3) was studied by Razouki^[8] and Razouki and Mohee^[9]. A series of six points on the axis of the first axle in the n-direction was chosen for this purpose. Table (1) shows that the investigation revealed that the maximum bending moment occurs at the center of one of the dual tires of the first axle (leading axle) as shown in Fig.3 for $d/L_R = 0.28$. Table (2) shows an investigation of the effect of the rotation on maximum bending moment. The investigation revealed that the worst condition of the rotation was for α (angle of rotation in degrees of the floating tandem axle subtended between the n-direction and the direction of travel) equals 0° . Accordingly, such a location and direction were adopted

throughout this work as the worst condition given the maximum moment due to the three-wheel assembly of floating tandem axle. For such a location, Fig.4 shows the variation of the ratio of bending moment to maximum bending moment with the angle of rotation α for case $d/L_R=0.280$ and for four values of total floating tandem axle loads of 100, 150, 200, and 250 kN.

COMPARISON BETWEEN WINKLER, FILONENKO-BORODICH AND REISSNER FOUNDATION MODELS

Fig.5 is constructed for comparison between Winkler model, Filonenko-Borodich, and Reissner model. It is obvious from this figure that the values of the moments for the case of Winkler foundation are much larger than the values of Reissner foundation and that the value of moments of Reissner model is less than that of the Filonenko-Borodich model. It is obvious from this figure that the variation in values of moments for the case of Winkler foundation is significantly higher than the variation in values of moments for the case of Filonenko-Borodich foundation. This is due to the fact of the effect of the membrane introduced between the spring elements and the foundation in Filonenko-Borodich model. Thus, and due to this difference in moment, the design based on Reissner foundation is much more economical than that based on Filonenko-Borodich foundation and than that based on Winkler foundation.

LOAD FACTOR CHARTS

For the purpose of developing the load factor charts for floating tandem axle loads, the equivalent single axle load (ESAL) which is twice the equivalent single wheel load (ESWL) should be determined first. Based on Reddy and Pranesh^[1] chart for interior loading on concrete pavement on Reissner foundation, the bending moment due to the three-wheel assembly of one side of the floating tandem axle (see Fig.2 and 3), can be obtained as follows:

$$M = qNL_R^2 / 10000 = FTAL \times L_R^2 \sum_{i=1}^{i=3} N_i / 6 \times 10000A \quad \dots(3)$$

where

M = bending moment at computational point due to floating tandem axle loads (N.m).

N_i = number of blocks enclosed on the chart within the contact area of the i^{th} tire.

A = contact area between pavement and one tire only (m^2).

FTAL= floating tandem axle load (ton.).

Adopting the equal maximum tensile stress criterion, the maximum bending moment due to the floating tandem axle load becomes equal to that due to the equivalent single axle load (ESAL). Thus, for equal contact area concept, equation (3) yields:

$$\begin{aligned}
 M &= FTAL \times L_R^2 \left(\sum_{i=1}^{i=3} N_i \right)_{\max.} / 60000A \\
 &= P_e \times L_R^2 \times N_e / 10000 \quad \dots(4) \\
 &= ESWL \times L_R^2 \times N_e / 10000A
 \end{aligned}$$

N_e = number of blocks enclosed within the contact area of the equivalent single wheel yielding maximum bending moment.

P_e = contact pressure = tire pressure (N/m^2) for the equivalent single wheel load where ESWL=equivalent single wheel load.

Accordingly,

$$ESWL / FTAL = \left(\sum_{i=1}^{i=3} N_i \right)_{\max.} / 6N_e \quad \dots(5)$$

Finally, by introducing the dimensionless load factor L_f as follows:

$$L_f = ESAL / FTAL, \quad \text{where}$$

L_f = load factor. ESAL= equivalent single axle load.

= $2 \times$ ESWL, the load factor becomes:

$$L_f = ESAL / FTAL = 2 \times ESWL / FTAL$$

$$= \left(\sum_{i=1}^{i=3} N_i \right)_{\max.} / 3N_e \quad \dots(6)$$

The worst location for the equivalent single wheel load yielding maximum bending moment is that in which the center of the contact area of one tire of the dual tire coincides with the origin of Reddy and Pranesh^[1] chart for Reissner foundation.

Fig.6 represents the load factor chart for floating tandem axle loads. The charts cover a range of floating tandem axle loads of 100kN to 250kN and a slab thickness range of 6in. (15.24cm) to 12in. (30.48cm). It is obvious from this figure that the load factor L_f increases with the increase in total floating tandem axle load and slab thickness.

STRESS CHART DEVELOPMENT

For the purpose of quick determination of the flexural stresses in concrete pavements due to floating tandem axle loads for interior loading, a stress chart is to be developed covering all practical ranges of the related parameters. The covered range for floating tandem axle loads for this chart is between 100kN and 250kN. Four values of concrete pavement slab thickness of 6in. (15.24cm), 8in. (20.32cm), 10in. (25.40cm), and 12in. (30.48cm) are considered. A unique floating tandem axle spacing of 130cm and a dual tire spacing of 30cm were adopted. Thus, the S/d ratio becomes 4.33. Fig.7 shows the stresses due to floating tandem axle load on the interior of concrete pavement on Reissner foundation. The maximum flexural stress was calculated using the following equation^[1,3] :

$$\sigma_{\max.} = 6 \times M_{\max.} / h^2$$

where

$\sigma_{\max.}$ = maximum flexural stress due to floating tandem axle

load on the interior of concrete pavement slab (N/m^2).

M_{\max} & h as defined before.

It is obvious from this figure that the stress on the interior of the concrete pavement slab increases with the increase in floating tandem axle load. It is also apparent from the figure that the maximum flexural stress decreases with the increase of slab thickness. The decrease is pronounced at the higher values of floating tandem axle loads than that at the lower ones.

CONCLUSIONS

The following conclusion can be drawn from this work.

1. The maximum bending moment and hence the maximum stress in the interior of concrete pavement slab due to floating tandem axle loads occurred at the center of each of the dual tires.
2. The load factor charts developed here can be used for direct determination of the equivalent single axle load for floating tandem axles acting on the interior of a concrete pavement slab for all practical cases of pavement characteristics and axle geometry.
3. The load factor is significantly affected by the floating tandem axle load magnitude. An increase in the floating tandem axle load causes an increase in the load factor. This effect is more pronounced at the lower values of floating tandem axle load than at the higher ones.
4. For a given floating tandem axle load, the load factor increases with increasing slab thickness and hence

increasing in relative stiffness of concrete pavement slab. Also this effect is more pronounced at the lower values of floating tandem axle load magnitude than at the higher ones.

5. For a given floating tandem axle load, the load factor increases with increasing slab thickness and hence increasing in relative stiffness of concrete pavement slab. Also this effect is more pronounced at the lower values of floating tandem axle load magnitude than at the higher ones.

6. The stress chart developed for floating tandem axles acting on the interior of the concrete pavement slab on Reissner foundation allows the direct determination of the maximum bending tensile stress in the slab for all practical purposes.

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Table (1): Effect of lateral shift of three-wheel assembly on the bending moment (case $d/L_R=0.28$, $S/d=4.33$, and $a/L_R=0.13$)

Point	Lateral Shift x/d	Angle of Rotation α	$M / M_{max.}$
1	0.00	0.00	1.000
2	0.20	0.00	0.8714
3	0.40	0.00	0.8643
4	0.60	0.00	0.8572
5	0.80	0.00	0.8072
6	1.00	0.00	0.8000

Table (2): Effect of rotation of three-wheel assembly on the bending moment (case $d/L_R=0.28$, $S/d=4.33$, and $a/L_R=0.13$)

Point	Lateral Shift x/d	Angle of Rotation α	$M / M_{max.}$
1	0.00	0.00	1.0000
2	0.00	15.00	0.9847
3	0.00	30.00	0.9756
4	0.00	45.00	0.9624
5	0.00	60.00	0.9431
6	0.00	75.00	0.9200
7	0.00	90.00	0.9015
8	0.00	100.00	0.8875

مخططات الإجهاد للمحاور المعلقة المزدوجة المؤثرة

على التبليط الكونكريتي المستند على أساس رايزنر

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الخلاصة

يعرض هذا البحث مخططات الإجهاد ومعامل الحمل للمحاور المعلقة المزدوجة المؤثرة على التبليط الكونكريتي الجالس على أساس رايزنر. يتألف المحور المعلق المزدوج من محور منفرد باطارين على كل جهة يتبع بمحور منفرد آخر بإطار واحد على كل جهة أو بالعكس. من المسح الهندسي لقوى المحاور تم الحصول على الميزات الهندسية للمحاور المعلقة المزدوجة وهي المسافة بين المحورين، المسافة بين الإطارين الجانبيين والموقع النسبي للمحور المنفرد. اعتماداً على نتائج المسح الهندسي تم الحصول على التوزيع التكراري لقوى المحاور وكذلك لضغط الإطارات. تم إقتراض مساحة التماس الدائرية بين إطار المحور المعلق المزدوج والتبليط الكونكريتي.

بالإستفادة من حل ريدي وبرانيش لعزوم الإنحناء في التبليط الكونكريتي على أساس رايزنر تم تعيين أقصى عزم إنحناء ناتج عن حمل المحور المعلق المزدوج. تم تطوير مخططات الإجهاد ومعامل الحمل للمحاور المعلقة المزدوجة لمختلف قيم سمك التبليط 6، 8، 10، 12 إنج (15، 24، 20، 32، 40، 25، 48، 30 سم) ولعدة قيم من الحمل الكلي الحقلي المسلط على المحور المعلق المزدوج. وبإستخدام هذه المخططات يتم الحصول مباشرة على أقصى عزم انحناء للتبليط الكونكريتي ولحمل محور معلق مزدوج وسمك تبليط معطيان.

الكلمات الدالة

التبليط الكونكريتي. أساس رايزنر. المحور المعلق المزدوج. مخططات الإجهاد. مخططات معامل القوة.