ELASTIC – PLASTIC NON – LINEAR BEHAVIOR OF SUDDENLY LOADED PLANE STEEL FRAMES BY ASI TECHNIQUE

Dr. Eyad K. S. Al-hachamee
Department of Building and Construction
University of the Technology

Assal T. Hussein
Researcher
Structural Engineering
Ministry of Construction and Housing

ABSTRACT

A non-linear finite element method for elastic – plastic analysis of steel plane frames under sudden loading is investigated. Geometric nonlinearity is considered using the Adaptively Shifted Integration (ASI) technique for the linear Timoshenko beam element.In (ASI) technique, the numerical integration point in an elastically deformed element is placed at the optimal point for linear analysis (midpoint in the linear Timoshenko beam), while the integration point is shifted immediately after the occurrence of a fully-plastic section in the element, using the previously established relation between the location of numerical integration point and that of plastic hinge, to form a plastic hinge exactly at the position of that section. Thus the technique produces higher computational accuracy with fewer elements than the conventional finite element code. As a result, the ASI technique gives a more precise solution with fewer elements than the conventional finite element method. .

The computer program used in this search employs the one – dimensional element with two nodes for steel structures. The major task of the present work is the development of a computer program which incorporates the above method of analysis, and the accuracy of the analytical results is assessed with respect to the available experimental and analytical data.

KEYWORDS

Elastic – Plastic Analysis, Geometrical Nonlinearity, Steel frame, Sudden load,

NOTATIONS

[K] : Stiffness Matrix

[M]: Consistent Mass Matrix

[D_p] : Stress-Strain Matrix for Plastic Deformation

[D_e] : Stress-Strain Matrix for Elastic Deformation

[B(s)]: Generalized Strain-Nodal Displacement Matrix

L : Length of Element

r : Location where stresses and strains are evaluated.

s: Location of the Numerical Integration Point.

Note: Other notations are described in the place of appearance

INTRODUCTION

The progressive collapse of the World Trade Center Towers in New York, USA, revealed the structural vulnerability of tall steel buildings to the impact of a fast moving object. Technical investigations are necessary to clarify the technical conditions that contributed to the progressive collapse and to improve the ways by which buildings are designed, constructed, maintained and used. Numerical simulation is considered to be one of the means to investigate such problems.[2]

The Finite Element Method (FEM), which is based on continuum material equations, has been successfully applied to a wide range of engineering problems including structural analyses of large-scale structures. Generally, it is difficult to determine the loads resulting in the structure due to dynamic load, and applying dynamic loads to an analytical model in the form of nodal forces may not simulate the dynamic phenomenon well. In this study, the formerly developed Adaptively Shifted Integration (ASI) technique for the linear Timoshenko beam element is used. The (ASI) technique gives a more precise solution than the conventional finite element method, and has become able to simulate dynamic behaviors with strong nonlinearities by a small number of elements. From the above, it is confirmed that the (ASI) technique is an efficient method for analyses of frame structures both in elastic and elasto-plastic ranges.[2]

Adaptively Shifted Integration (ASI) technique

Elasto-plastic analyses of framed structures using the linear Timoshenko beam element are based on the following incremental form of the principle of virtual work [2]:

$$\delta V - \delta W = \int_{-L/2}^{L/2} \{\delta\Delta\varepsilon\}^T \{\Delta\sigma\}dz - \{\delta\Delta u\}^T \{\Delta f\} = 0 \qquad \text{….(1)}$$

In Eq.(1), δV and δW are the internal work (strain energy)and the external work respectively. $\{\Delta\varepsilon\}$, $\{\Delta\sigma\}$, $\{\Delta u\}$,and $\{\Delta f\}$ are the generalized strain increment vector, the generalized stress (resultant force) increment vector, the nodal displacement increment vector and the external force increment vector, respectively. The symbols **δ** and **Δ** denote variation and increment.

The relation between the generalized strain increment and the nodal displacement increment vectors are given by the following equation:

$$\{\Delta\varepsilon\{r)\} = [B(s)]\ \{\Delta u\} \qquad \text{….(2)}$$

Here, $[B(s)]$ is the generalized strain-nodal displacement matrix. **s** is the location of the numerical integration point and **r** the location where stresses and strains are actually evaluated. **s** is a non-dimensional quantity, which takes a value between –1 and 1. It should be noted here that the linear Timoshenko beam element uses one-point integration to avoid shear locking.

When the element behaves elastically, the relation between resultant force increment vector and generalized strain increment vector is given by the following equation:

$$\{\Delta\sigma(r)\} = [D_e(r)]\,\{\Delta\varepsilon\{r)\} \qquad \text{….(3)}$$

Here, $[D_e]$ is the stress-strain matrix for elastic deformation. [De] will change to $[D_p]$, when plastic deformation is determined by the plastic flow theory using the following form of yield function:

$$f_y(\sigma(r)) - 1 = 0 \qquad \text{….(4)}$$

Substitution of Eqs.(2) and (3) into Eq.(1) leads to the following form of element stiffness matrix:

$$[K] = L[B(s)]^T[D_e(r)][B(s)] \qquad \text{….(5)}$$

In the ASI technique, numerical integration points are shifted immediately after the formation of a fully plastic section in the element in order to form a plastic hinge exactly at that section. When a plastic hinge is judged to be unloaded, the corresponding numerical integration point is shifted back to its normal position. Here, the normal position means the location where the numerical integration point is placed when the element acts elastically.

Figure (1) shows a linear Timoshenko beam element and its physical equivalence to a rigid body-spring model (RBSM). As shown in Figure (1) the relationship between the locations of the numerical integration point and the stress evaluation point where a plastic hinge is actually formed is expressed as follows:

$$r_1 = - s_1 \qquad \text{….(6a)}$$

$$s_1 = - r_1 \qquad \text{….(6b)}$$

When the entire region of an element behaves elastically, the numerical integration point is placed at the midpoint of the element ($\mathbf{s_1}=0$) which gives $\mathbf{r_1}=0$ from Eq.(6) in this case, the elemental stiffness matrix for the element is given by the following equation**[2]**:

$$[K] = L[B(0)]^T[D_e(0)][B(0)] \qquad \text{….(7a)}$$

The generalized strain increment vector $\{\Delta\varepsilon\ (0)\}$ and the resultant stress increment $\{\Delta\sigma\ (0)\}$ are calculated as follows [2]:

$$\{\Delta\varepsilon\ (0)\} = [B(0)]\{\Delta u\} \qquad \text{….(7b)}$$

$$\{\Delta\sigma\,(0)\} = [D_e(0)]\,\{\Delta\varepsilon\,(0)\} \quad \text{….(7c)}$$

After the formation of a fully plastic section, the numerical integration point is shifted immediately to a new point (**$s_1 = -r_1$**). For instance, if a fully plastic section occurs at the right end of an element (**r_1**$= 1$), the numerical integration point is shifted to the left end (**s_1**$= -1$) and vice versa. In this case, the element stiffness matrix, the generalized strain and resultant stress increment vectors are given as follows [2]:

$$[K] = L[B(s_1)]^T[D_p(r_1)][B(s_1)] \quad \text{….(8a)}$$

$$\{\Delta\varepsilon\,(r_1)\} = [B(s_1)]\{\Delta u\} \quad \text{….(8b)}$$

$$\{\Delta\sigma\,(r_1)\} = [D_p(s_1)]\,\{\Delta\varepsilon\,(r_1)\} \quad \text{….(8c)}$$

Here, $[D_p]$ is the stress-strain matrix for plastic deformation.

In this study a computer program has been built using the above equation for elastic – plastic analysis for a steel plane frame and the F.E.M. with ASI technique applied to updating the equation of motion for each step of applied dynamic load. Boundary conditions of free vibration behavior are considered in this study.

Numerical Examples

1 Example 1

A steel plane frame with properties shown in Figure (2) is subjected to a dynamic load 100 kip(444.8 kN), this is used as an input data for the computer program. The results of the computer program, Figure (3) shows the elastic – plastic response of the plane frame by using (ASI) technique. Figure (4) shows the time sequence of occurrence of plastic hinges.

2 Parametric Study

The main purpose of the present section is to investigate the effect of several important parameters on the behavior of a simple plane frame subjected to dynamic loads by using the (ASI) technique. The frame described in section (3.1) is used to perform this investigation.

2.1 Accuracy of ASI Technique

Figure (5) shows the elastic – plastic response of a plane frame using both (ASI) technique and the conventional F.E.M. These results show that the one – element per member by the (ASI) technique is enough to fit the results, compared with 32 elements per member approximation by the conventional F.E. method.

2.2 Time Step

Figure (6) shows the elastic – plastic response of a plane frame for various time step lengths. The response indicates that a good accuracy can be obtained even with large values of time step length.

2.3 Mass of The Structure

Figure (7) shows the elastic – plastic response of a steel plane frame for various cross sections of the structure .It is observed that when the mass of the structure increases, the displacement response decreases.

2.4 Initial Velocity of Applied Load

Figure (8) shows the elastic – plastic response of the steel plane frame for various initial velocities of the dynamic load .It is observed that when the initial velocity increases, the displacement response increases.

2.5 Converged Solution

Figure (9) shows the elastic – plastic response of the plane frame for one, two, and four elements per memeber.It is observed that the (ASI) technique gives results with high accuracy at very low calculation cost even when the member of elements per member is very low.

CONCLUSIONS

1. In the elastic analysis there is no distinction between the ASI technique and the conventional finite element method.

2. This investigation has shown that the Adaptively Shifted Integration (ASI) technique for elastic – plastic nonlinear analysis under dynamic load is capable of predicting with reasonable accuracy the behavior of steel frame structures.
3. The ASI technique is able to cope with dynamic behaviors with strong nonlinearities including member fracture .
4. Strong nonlinearity problems, in which complicated analysis processes are needed in the conventional finite element analysis, can be easily analyzed by implementing the ASI technique to the finite element codes utilizing the linear Timoshenko beam element foe skeletal (frame) structures.
5. The one – element per member solution by (ASI) technique is compared with the results of 32 -elements per member approximation searched by the conventional finite element method. The (ASI) technique is more economical.
6. The mass of the structure was found to be an important parameter in determining the elastic – plastic response of steel plane frames. It is observed that when the mass of the structure increases 3.87 %, the displacement response decreases (0.5 % - 0.6 %).
7. The initial velocity of dynamic load was found to be also an important parameter in determining the elastic – plastic

response of steel plane frames. It is observed that when the initial velocity increases the displacement response increases.

8. In the elastic - plastic analysis, it was found that the (ASI) technique needs not more than two elements per member to get an accurate solution, more elements may not refine the solution.

9. The (ASI) technique gives results with high accuracy at very low calculation cost as the needed number of elements per member is very low.

REFERENCES

1. Newmark N. M., “A Method of Computation for Structural Dynamics” Journal of Engineering Mechanics Division, ASCE, Vol. 85, No.EM3, pp. 67-94,(1959).

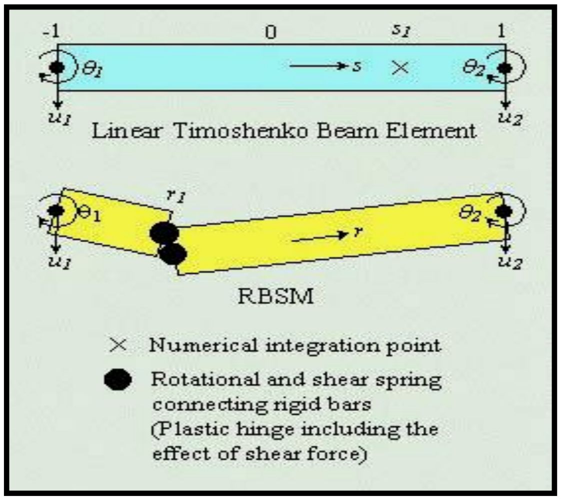
2. Lynn, K. M. and Isobe, D., “Structural Collapse Analysis of Framed Structures under Impact Loads Using ASI-Gauss Finite Element Method”, Summaries of Technical Papers of Annual Meeting, Architectural Institute of Japan 2003, B-1, pp.333-334, (2003).

3. Toi, Y. and Isobe, D. “Finite Element Analysis of Quasi – Static and Dynamic Collapse Behaviors of Framed Structures by the

Adaptively Shifted Integration Technique", J. Computers & Structures Vol.58, No. 5, pp. (947-955), (1996).

4. Biggs, J. M., "Introduction to Structural Dynamics", McGrew-Hill Book Co., New York, 1964.

5. Ross, C.T.F., "Finite Element Techniques In Structural Mechanics", Albion Publishing, Chichester, (1996).



Figure(1) LinearTimoshenko beam element and its physical equivalent (2)

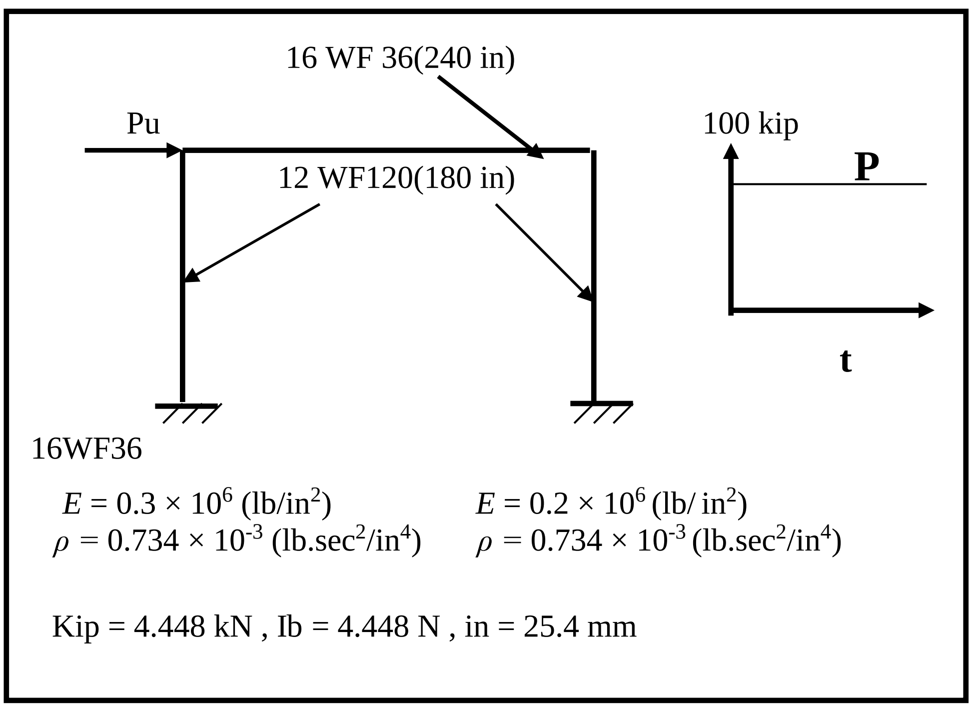


Figure (2) Properties of steel plane frame

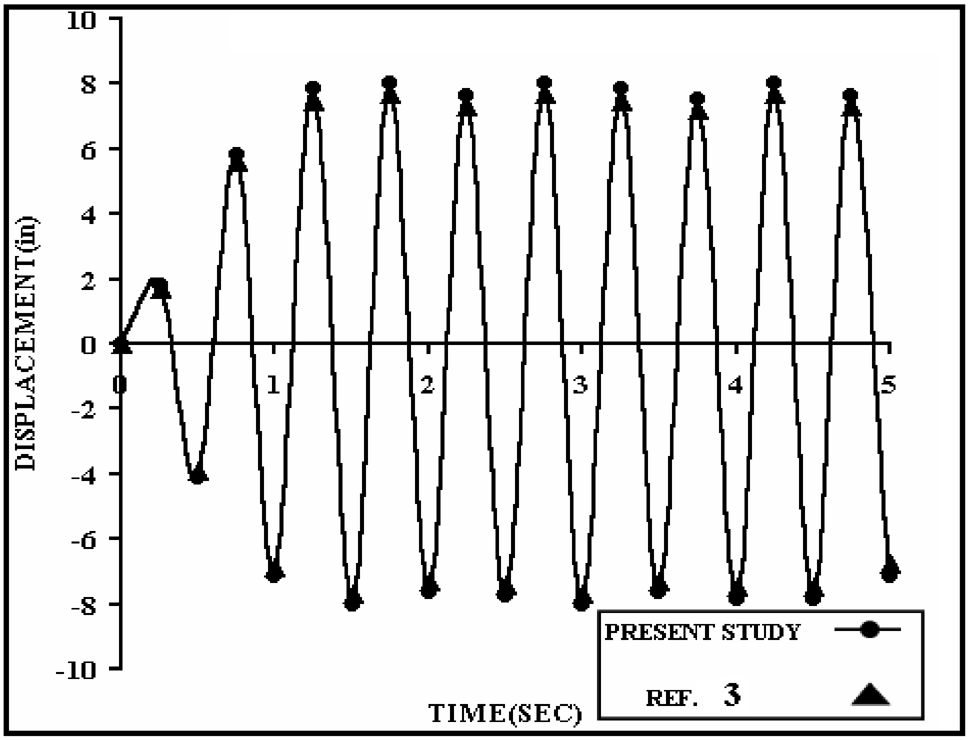


Figure (3) Elastic – Plastic response for steel plane frame

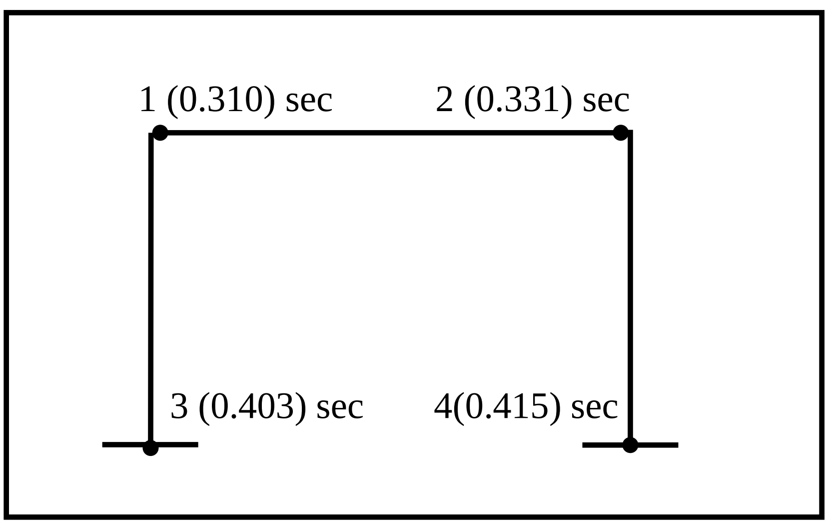


Figure (4) Time sequence of occurrence of plastic hinge

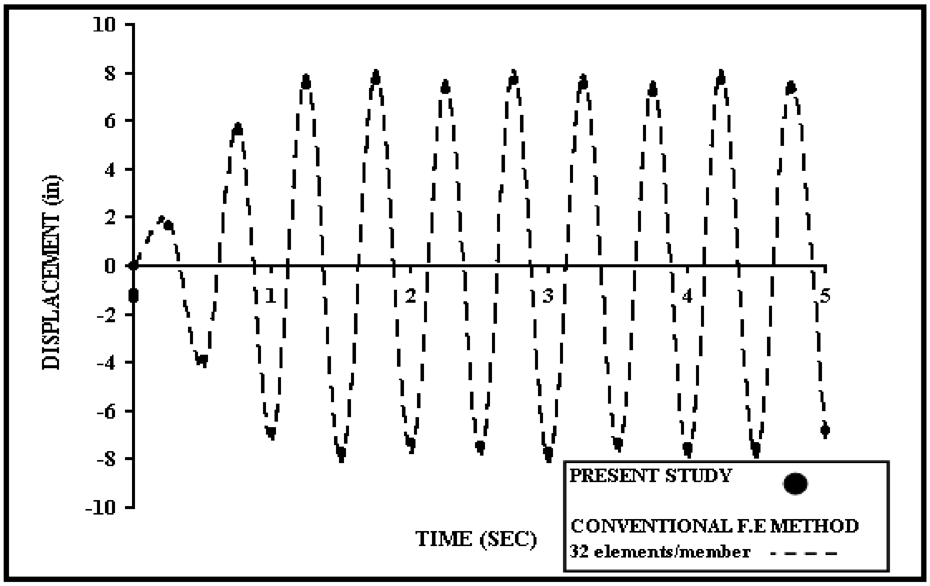


Figure (5) Comparison between F.E. ASI technique and conventional F.E.M.

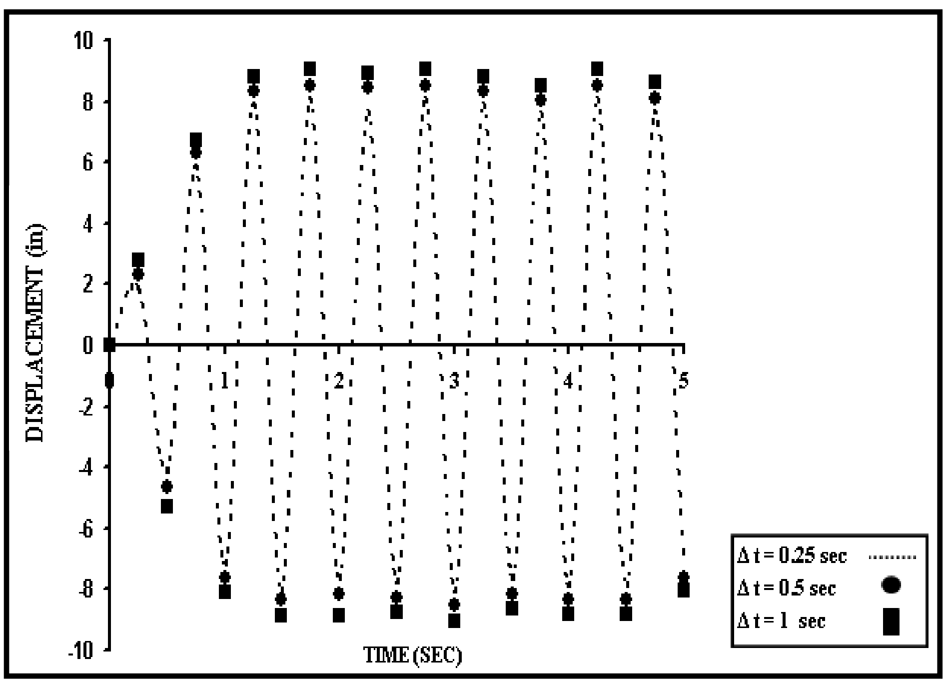


Figure (6) Elastic – plastic response for steel plane frame with various time steps

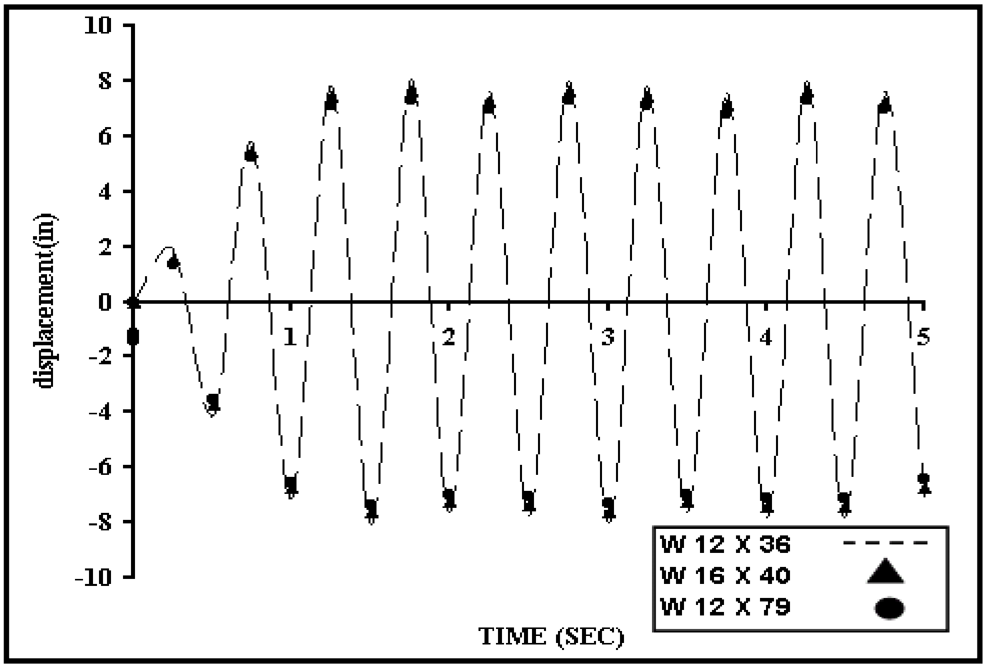


Figure (7) Effect of mass of structure on elastic – plastic response for steel plane frame

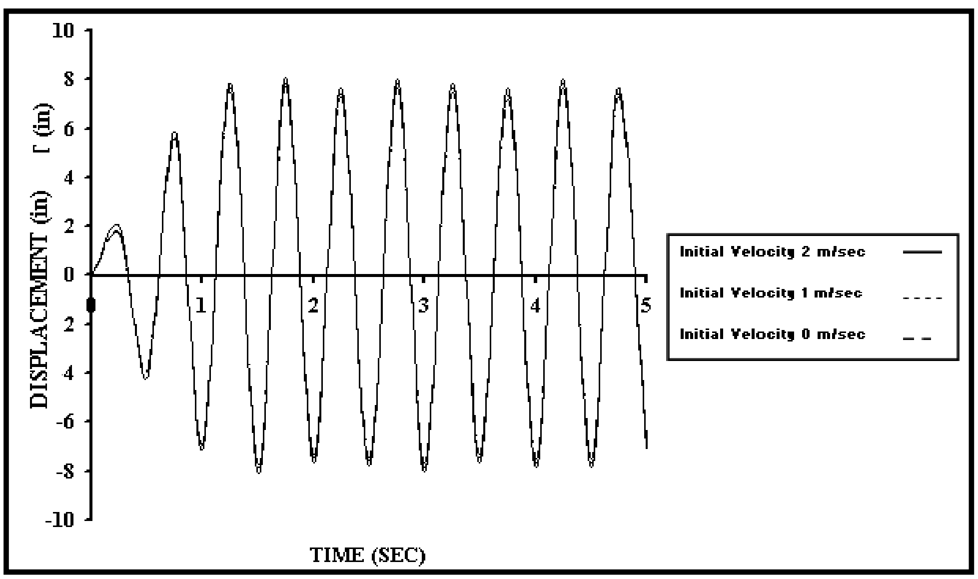


Figure (8) Elastic – plastic response of steel plane frame with various initial velocity

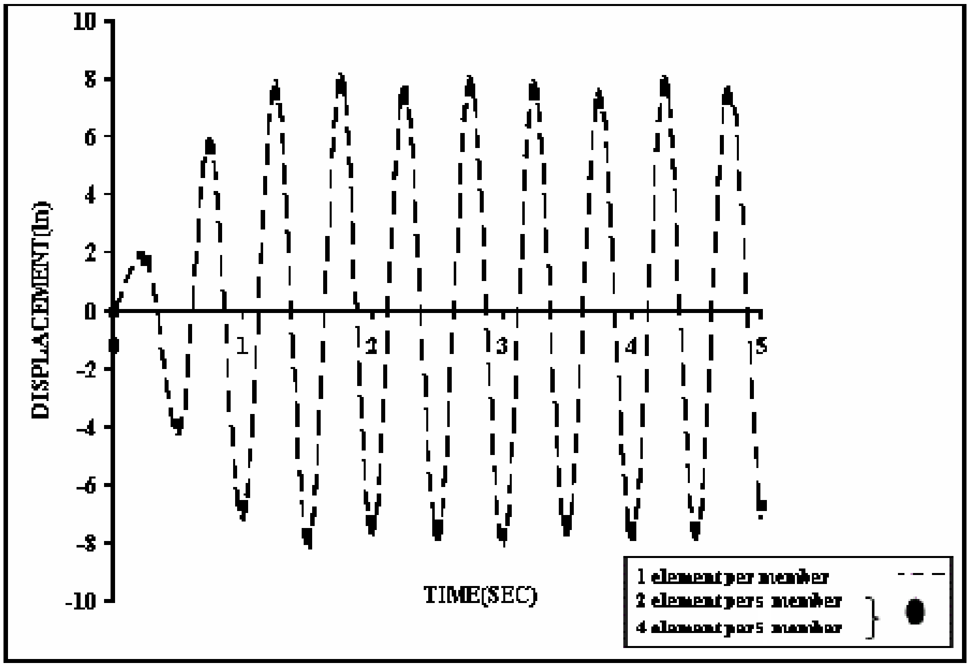


Figure (9) converged solution for ASI technique even when using a low number of elements per member

السلوك المرن – اللدن للهياكل الحديدية المستوية تحت أحمال مفاجئة بطريقة (ASI)

د. أياد كاظم صيهود الحجامي
قسم هندسة البناء والانشاءات
الجامعة التكنولوجية

أسل تحسين حسين – باحث
وزارة الاسكان والتعمير

الخلاصة

في هذا البحث تم دراسة التصرف المرن – اللدن اللاخطي للأبنية الحديدية المستوية المعرضة للأحمال المفاجئة بأستخدام طريقة العناصر المحددة. وقد أخذت بنظر الأعتبار التغيرات الشكلية بأعتماد تقنية (ADAPTIVELY SHIFTED INTEGRATION) للعنصر (LINEAR TIMOSHENKO BEAM ELEMENT) . في طريقة(ASI) يتم حساب التشوهات المرنة فيها بأستخدام طريقة التكامل العددي بالنسبة للتحليل الخطي للعقد المثالية (في منتصف العنصر) , وعند حصول تشوه لدن كامل للمقطع فان العقدة تحول مباشرة الى النهاية الأخرى للعنصر ,وتستعمل العلاقة بين موقع عقدة التكامل العددي وموقع المفصل اللدن لتشكيل مفصل لدن في ذلك المقطع . هكذا فان هذه التقنية تُنتجُ دقةَ حسابيةَ أعلى بعدد أقل من العناصر فيما لو تم استخدام الطريقة التقليدية المستخدمة في نظرية العناصر المحددة. كنتائج فان هذه التقنية في الحل بعناصر اقل تَعطي دقّةً حَلّ أكثر مِنْ النظرية التقليديِة . البرنامج ألحاسوبي المستخدم في هذا البحث استخدم العنصر أحادي البعد ذو العقدتين للمنشآت الحديدية ,إنّ الهدفَ الرئيسيَ للدراسة الحالية تطوير برنامج حاسوبِ يدعم الطريقةَ أعلاهَ في التحليلِ. ولبيان دقةَ النَتائِجِ التي تم الحصول عليها تمت مقارنتها بالبياناتِ التجريبيةِ والتحليليةِ المتوفرة وقد أظهرت نتائج جيدة.

الكلمات الدالة

التحليل المرن – اللدن ، الاخطية الهندسية ، هيكل حديدي ، حمل فجائي