Tikrit Journal of Eng. Sciences/Vol.13/No.3/October 2006

(34-53)

ECONOMICAL DESIGN OF CORBELS

Hasan Jasim Mohammad Al-Badri

Lecturer

Civil Eng. Dept.- Tikrit University

ABSTRACT

This study is an application of optimization method to the structural design of corbels, considering the total cost of the corbel as an objective function with the properties of the corbel and shear span, dead load, live load and corbel width, as design variables.

A computer program has been written to solve numerical examples using the ACI code equations and all new requirements and criteria in concrete design.

It has been proved that the minimum total cost of the corbel increases with the increase of the shear span, and decreases with the increase of the friction factor for monolithic construction.

NOTATIONS

A_c effective concrete area *(bd)*.

A_h area of hoop stirrups reinforcement.

A_f area of flexural steel reinforcement.

A_n area of steel reinforcement for tensile force.

A_s total steel reinforcement .

A_{vf} total shear-friction reinforcement.

M_n nominal bending moment.

M_u ultimate bending moment.

V_n nominal vertical reaction (shear).

V_u ultimate vertical reaction (shear).

N_{uc} tensile horizontal force.

a shear span

b supporting column width

d effective depth.

f_c' concrete compressive strength.

f_y yield strength of steel.

h depth of corbel at face of column.

Φ reduction factor.

μ friction factor for monolithic construction.

INTRODUCTION

Brackets (or Corbels) such as shown in Fig. (1) are widely used in precast construction for supporting precast beams at the columns. When they project from a wall, rather than from a column, they are properly called corbels, although the two terms are often used interchangeably. Brackets are designed mainly to provide for the vertical reaction V_u at the end of the supported beam, but unless special precautions are taken to avoid horizontal forces caused by restrained shrinkage, creep (in the case of prestressed concrete beams), or temperature change, they must also resist a horizontal force N_{uc}[1].

PURPOSE OF STUDY

The purpose of this study is to detect the capabilities of optimization method to handle the economical structural design of a corbel. Giving a safe design with minimum cost based on considering the effects of different parameters on the corbel and giving the designer the relationships and curves between design variables, the design of a corbel can be more economical,reliable and simple.

HISTORICAL BACKGROUND

Torres et al., (1966),as reported by Al-Jubori (2001) presented the minimum cost design of prestressed concrete highway bridges subjected to AASHTO loading by using piecewise LP (load program) method [2].

Kirsch (1972) presented a minimum cost of a continuos two-span prestressed concrete beam .The cost function included only the cost of concrete and the cost of prestressing steel [3].

Namman (1982) presented a minimum cost design of prestressed concrete tension member based on the ACI-Code 1977 .The cost function included the material costs of concrete and the prestressing steel [4].

Al-Jubair (1994) minimized the cost of ring foundations by using the simplex method of Nelder and Mead. The results obtained supported the efficiency of optimization techniques in selecting the most economical design of ring foundations for given conditions [5].

Al-Douri (1999) minimized the cost of rectangular combined footings by using several methods .She concluded that the minimum cost of the footing decreases with increasing the distance between the columns for a constant length [6].

Al-Jubori (2001) minimized the cost design of mat foundations. He proved that the minimum cost of the raft foundation decreases with increasing of the angle of internal friction of the soil and increases with increasing the column spacing in both directions as well as with increasing the difference between the loads of adjacent columns[2].

OBJECTIVE FUNCTION

The total cost of a corbel can be represented by:

$$ZT=CSRE+CSFW+CSCO \qquad \ldots\ldots(1)$$

where:

ZT= Total cost (unit price).

CSRE= Cost of corbel reinforcement (unit price).

CSFW= Cost of corbel formwork (unit price).

CSCO= Cost of corbel concrete (unit price).

CSRE= ReTo*COR

$$=[A_s/(\pi/4*D_1^2)+A_h/(\pi/4*D_2^2)]*1.4*COR \quad \ldots(2)$$

CSFW= AMR*COFW

$$=\{[(h'+h)(a+50)]+[\sqrt{2}(h-h')b]+[h.b+2h_c.h]\}*COFW \quad \ldots(3)$$

CSCO= VMR*COCO

$$=[(h'+h)/2*(a+50)b+h_c.h.b]*COCO \quad \ldots(4)$$

and where:

TOTRE= Total amount of reinforcement steel (ton

COR= Price of reinforcement (unit price/ton)

COFW= Price of formwork (unit price/m^2)

COCO= Price of concrete (unit price/m^3)

A_s,A_h,D_1,D_2 = Area of main steel reinforcement, area of hoop bars, diameter of main steel and diameter of hoop steel respectively.

h´,h,a = Dimensions of corbel.

b, h_c = Dimensions of supporting column.

STRUCTURAL FORMALATION

The structural performance of a corbel can be visualized easily by means of the strut-and-tie model shown in Fig. (1). The downward thrust of the load V_u is balanced by the vertical component of the reaction from the diagonal compression concrete strut that carries the load down into the column. The outward thrust at the top of the strut is balanced by the tension in the horizontal tie bars across the top of the corbel; these also take the tension, if any, imparted by the horizontal force N_{uc}.

The steel required, according to the strut-and-tie model, is shown in Fig. (1) The main bars A_s must be carefully anchored because they need to develop their full yield strength f_y directly under the load V_u, and for this reason they are usually welded to the underside of a bearing angle at the load side. A 90° hook is provided for anchorage at the other side. Closed hoop bars with area A_h confine the concrete in the two compression strut and resist a tendency for splitting in a direction parallel to the thrust.

The provisions of ACI Code 11.9 for the design of brackets and corbels have been developed mainly based on tests [1] and relate to the flexural model of bracket behavior. They apply to brackets and corbels with a shear span ratio a/d of 1.0 or less (see Fig. 1). The distance d is measured at the column face, and the depth at the outside edge of the bearing area must not be less than 0.5d. The usual design basis is employed, i.e., $M_u \leq \Phi M_n$ and $V_u \leq \Phi V_n$, and for brackets and corbels (for which shear dominates the design),Φ is to be taken equal to 0.85 for all strength calculations, including flexural and direct tension as well as shear.

The section at the face of the supporting column must simultaneously resist the shear V_u, the moment $M_u = V_u.a$, and the horizontal tension N_{uc}. Here a is the shear span (or arm). Unless special precautions are taken, a horizontal tension not less than

20 percent of the vertical reaction must be assumed to act. This tensile force is to be regarded as live load, and a load factor of 1.6[7]should be applied.

An amount of steel A_f to resist the moment M_u can be found by the usual methods for flexural design. Thus,

$$A_f = \frac{M_u}{\Phi f_y(d-a/2)} \qquad \ldots(5)$$

where

$a = A_f f_y / 0.85 f_c' b$

is the depth compressive stress block. An additional area of steel A_n must be provided to resist the tensile component of force:

$$A_n = \frac{N_{uc}}{\Phi f_y} \qquad (6)$$

The total area required *for flexural and direct tension* at the top of the bracket is thus:

$$A_s \geq A_f + A_n \qquad (7)$$

Design for shear is based on the shear-friction method of Sec.4.10[1], and the total shear-friction reinforcement A_{vf} is found by:

$$A_{vf} = \frac{V_u}{\Phi \mu f_y} \qquad (8)$$

where the friction factor μ for monolithic construction is 1.40 for normal weight concrete, 1.19 for " sand-lightweight " concrete, and 1.05 for " all-lightweight " concrete. The usual limitations that Vn= Vu/Φ must not exceed the smaller of 0.2 $f_c'A_c$ or $800A_c$ apply to the critical section at the support face. Then, according to ACI Code 11.9, the total area required for shear plus direct tension at the top of the bracket is

$$A_s \geq (2/3)A_{vf} + A_n \qquad \ldots(9)$$

with the remaining part of A_{vf} placed in form of closed hoops having area A_h in the lower part of the bracket, as shown in Fig. (1).

Thus, the total area A_s required at the top of the bracket is equal to the larger of the values given by Eq. (7) or Eq. (9). An additional restriction, that A_s must not be less than 0.04(fc´/ fy) bd, is intended to avoid the possibility of sudden failure upon formation of a flexural tensile crack at the top of the corbel.

According to the ACI Code, closed hoop stirrups having area A_h (see Fig. 1) not less than $0.5(A_s\text{-}A_n)$ must be provided and be uniformly distributed within *two-thirds* of the effective depth adjacent to and parallel to A_s. This requirement is more clearly stated as follows:

$$A_h \geq 0.5A_f \quad \text{and} \quad \geq 1/3A_{vf} \qquad (10)$$

COMPUTER PROGRAM

The main program, utilized to perform the necessary calculations for optimization, was drawn from Bundy (1984) [8] and translated to FORTRAN-77.Hooke and Jeeves method was used to performed the minimization process utilizing this method of solution. Followings are the required input parameters for this program.

Ns- number of independent (design) variables.

X(Iz)-initial estimate of the design variables [Iz=1,2,3,……Ns]

Hz-step length.

The program (Corbel .For) in FORTRAN-77 is written by using the design procedure of ACI-Code with code improvement in load factors [7]. This program gave good results with code requirements and other design criteria.

The program (Corbel .For) uses a subroutine with the program (H & J. For). Input data symbols and other parameters used in subroutine (Corbel .For) is listed in Table (1) and results shown in Table (2).

NUMERICAL EXAMPLE

The basic data of the problem is shown in Fig. (2) .The problem was solved by using three initial trial values for design variables vector X=[a, V_d, V_l, b] .The input data is: Ns=4. The first initial trial values: X(1)=140 , X(2)=111 , X(3)=227 , X(4)=300. The second initial trial values: X(1)=175 , X(2)=111 ,

X(3)=227 , X(4)=350.The third initial trial values: X(1)=250, X(2)=120 , X(3)=250 , X(4)=350 .
Hz=0.01

The results obtained are shown in Table (3). Figs (3) to (5) show the convergence rate towards the minimum cost design of corbel.

DISCUSSION OF RESULTS

A parametric study was done to the shear span, corbel width, and friction factor for monolithic construction for the first initial trial point. The results are listed in Tables (4),(5) and (6).

It can be observed from Table (4) and Figs. (6) to (9) that as the shear span increases; the minimum total cost is increased, Fig (6). The increase is noticed after a shear span value of 150mm, and minimum total cost is at 140mm. The optimum dead load and live load are slightly decreased after a shear span 150mm, Figs. (7) to (8). The optimum corbel width is increased after the shear span of 150mm, Fig. (9).

It can be observed from Table (5) and Figs (10) to (13) that as the corbel width increases; the minimum total cost decreases then increases, Fig. (10). The optimum shear span is decreased, Fig. (11). But the optimum dead load and live load are increased Figs. (12) and (13).

It can be realized from Table (6) and Figs (14) to (18) that as the friction factor for a monolithic construction increases; the minimum total cost is decreased when concrete is of normal weight type. The optimum shear span, dead load, live load, and corbel width are increased when the friction factor is increased.

CONCLUSIONS

1-The economical structural corbel design can be handled as a problem of mathematical programming.

2-Optimization techniques are powerful to be applied to the optimum structural corbel design.

3-The minimum total cost is more sensitive to the changes in shear span and corbel width.

4-Increase in shear span leads to increase in minimum total cost and corbel width.

5-Increase in corbel width leads to increase in minimum total cost. So , increases are obtained in dead load and live load .

6-Increase in friction factor leads to decrease in total cost and increases in shear span, dead load, live load, and corbel width.

REFERENCES

1-Nilson, A. H. ‘’ Design of Concrete Structures ’’ , 12 th Ed. The McGraw-Hill Companies, In., (1997).

2-Al-Jubori , A. M. , “ Optimum Design of Raft Foundations ”, M.Sc. Thesis ,Tikrit University , College of Engineering, Civil Engineering Department (2001).

3 -Kirsch , U. “ Optimum Design of Prestressed Beams “,

Computers and Structures , Vol.2No.4 , pp. 573-583 (1972) .

4-Naaman , A. E. , " Optimum Design of Prestressed Concrete Tension Members " , ASCE , Vol.108 , No. 8 ,pp. 1722-1738 (1982).

5-Al-Jubair, H.S. , '' Economical Design of Ring Foundations '' Al-Muhandis ,Vol.120,No.4, pp.45-54(1994).

6-Al- Douri , E . M. ,'' Optimum Design of Rectangular Combined Footings'' , M.Sc. Thesis, Department of Civil Engineering, Tikrit University (1999).

7- ACI Committee 318-02,'' Building Code Requirements for Structural Concrete'' , ACI, Detroit (2002).

8-Bundy , B. D. , '' Basic Optimization Methods ' , Edward Arnold Publishers (1984).

9-ACI Committee 318-89 , '' Building Code Requirements for Reinforced Concrete'', ACI, Detroit (1989).

Table (1) Some Input Data

Symbols	Value	Function
a	140	Shear span (mm)
V_d	111	Vertical dead load (kN)
V_l	227	Vertical live load (kN)
b	300	Corbel width (mm)
SYS	414	Yield of steel strength (MPa)
CS	34.5	Concrete compressive strength (MPa)
MUO	1.4	Friction factor for monolithic construction

Table (2) Some Results of (Corbel .For)

	Corbel. For	Ref.[1]
A_s (mm^2)	975	966
A_h(mm^2)	343	342

Table (3) The Design Results (initial trial point)

Variables	First trial	Second trial	Third trial
Cost (U.P.)	2522634	3046880	5379943
a (mm)	137.55	165.75	237.65
V_d (kN)	105.10	102.40	112.30
V_l (kN)	220.50	219.25	240.02
b (mm)	290.10	330.20	340.00
FE *	368	375	384

* Number of function evaluation.

Table (4) The Design Results for different shear spans

Variables(mm)	a=140	a=150	a=180	a=190	a=200
Cost (U.P.)	2522634	2715063	7981240	8599066	9561101
V_d (kN)	105.10	51.50	51.00	34.50	43.00
V_l (kN)	220.50	167.55	167.00	150.50	159.00
B (mm)	290.10	284.50	304.50	322.00	332.00
A_s (mm^2)	922	644	641	501	639
A_h(mm^2)	324	227	226	176	232
FE*	368	287	193	222	208

* Number of function evaluation.

Table (5) The Design Results for different corbel width

Variables(mm)	b=250	b=300	b=350	b=400	b=480
Cost (U.P.)	4695887	2522634	3457989	7741330	5078185
A (mm)	108.00	105.10	94.00	94.00	93.00
V_d (kN)	160.00	220.50	227.00	277.00	290.00
V_l (kN)	242.00	290.10	239.00	311.00	360.00
A_s (mm^2)	501	922	724	642	863
A_h(mm^2)	277	324	255	226	324
FE*	280	368	351	351	355

* Number of function evaluation.

Table (6) The Design Results for different friction factor

Variables	μ=1.40	μ=1.19	μ=1.05
Cost (U.P.)	2522634	3034573	3034571
Shear arm a (mm)	137.55	135.25	133.30
V_d (kN)	105.10	101.20	98.20
V_l (kN)	220.50	210.30	202.50
b (mm)	290.10	285.20	280.00
A_s (mm^2)	922	862	798
A_h(mm^2)	324	330	340
FE*	368	350	342

* Number of function evaluation.

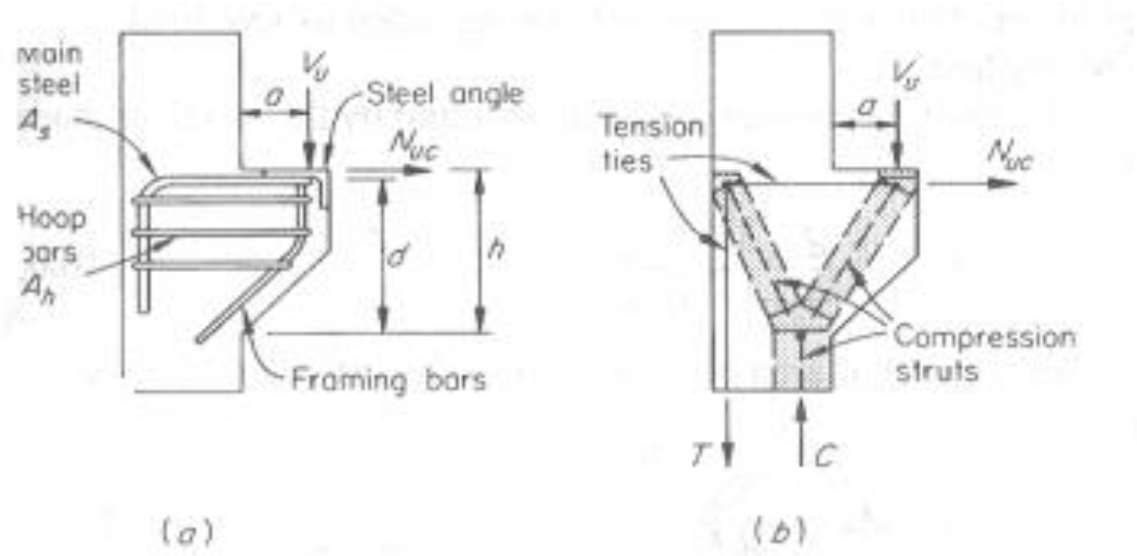

Fig. (1) Typical reinforced concrete corbel(a) Loads and reinforcement (b)Strut-and-tie model for internal forces

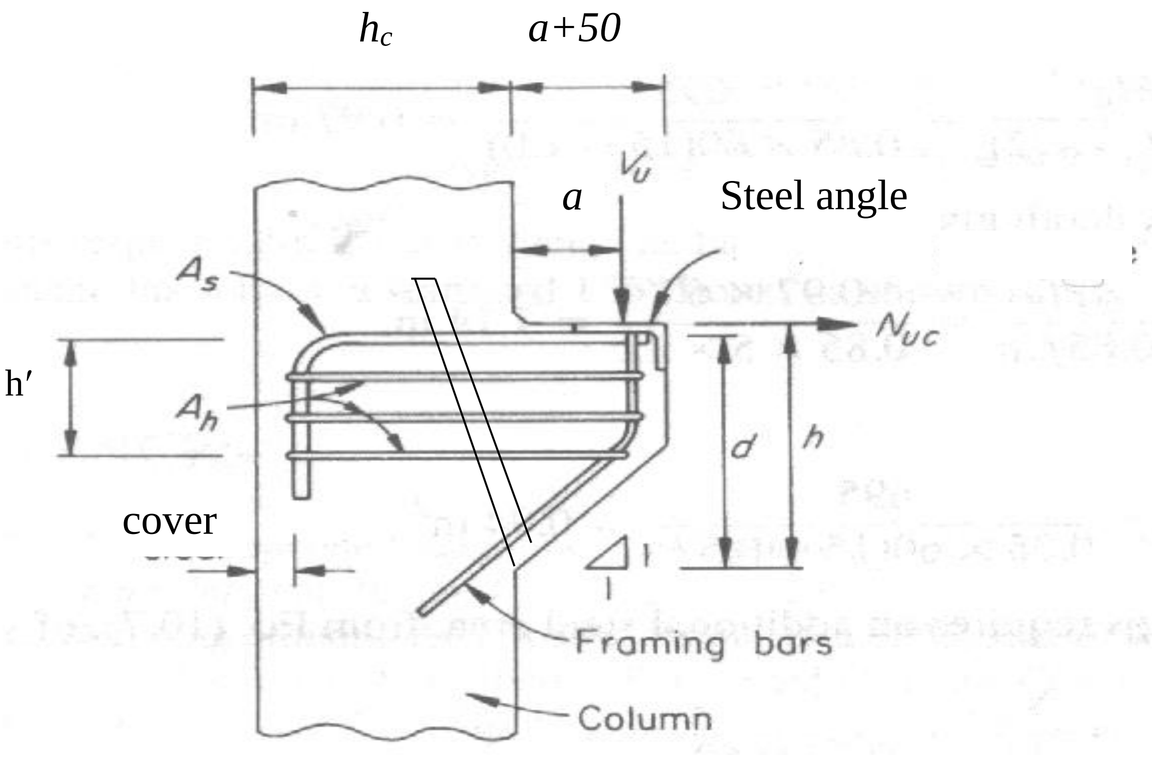

Fig. (2) Column bracket design example

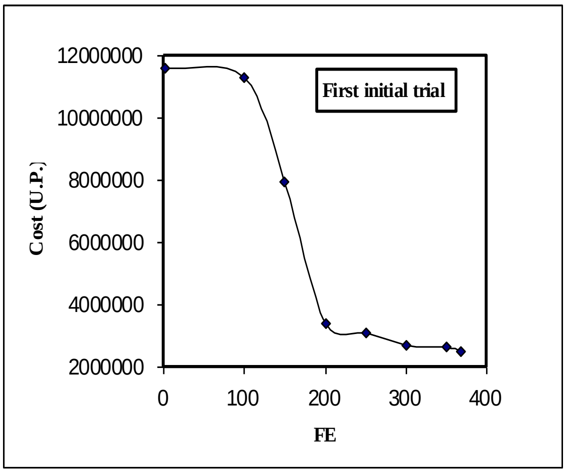

Fig. (3) Convergence towards the minimum cost

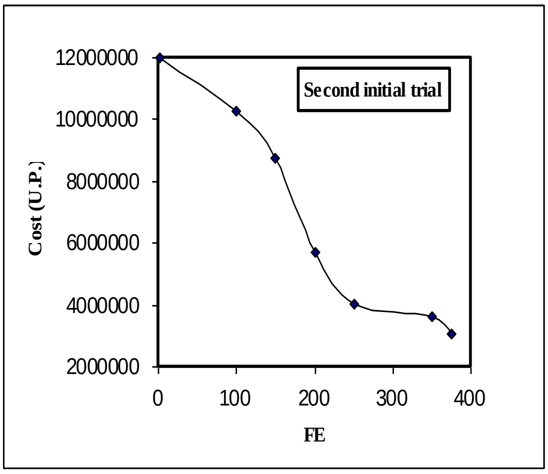

Fig. (4) Convergence towards the minimum cost

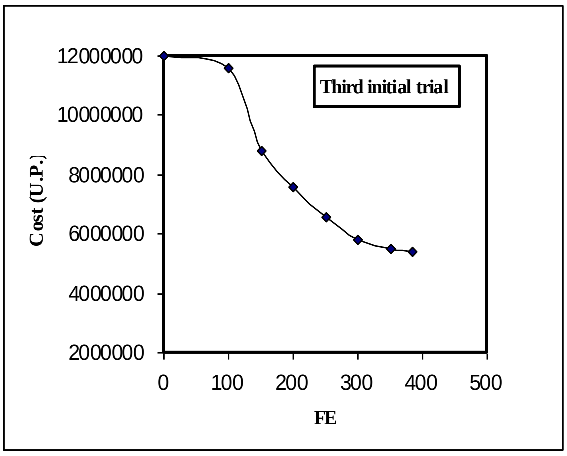

Fig. (5)Convergence towards the minimum cost

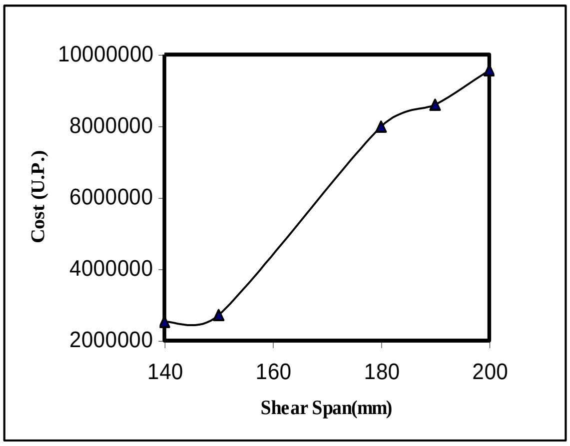

Fig. (6) Minimum total cost vs. shear span

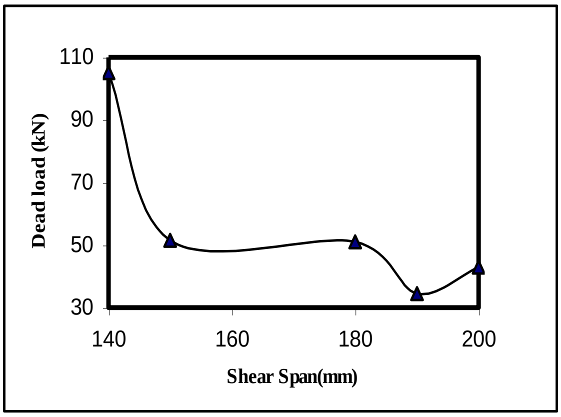

Fig. (7) Optimum dead load vs. shear span

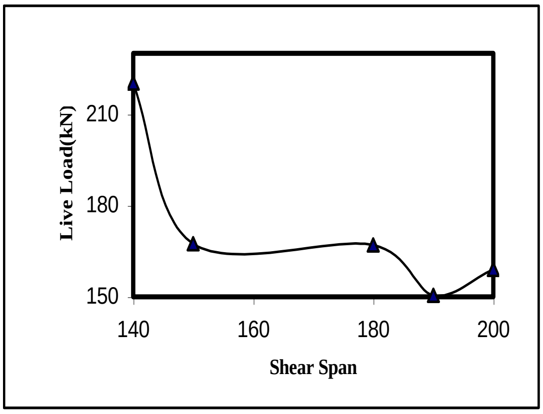

Fig. (8) Optimum live load vs. shear span

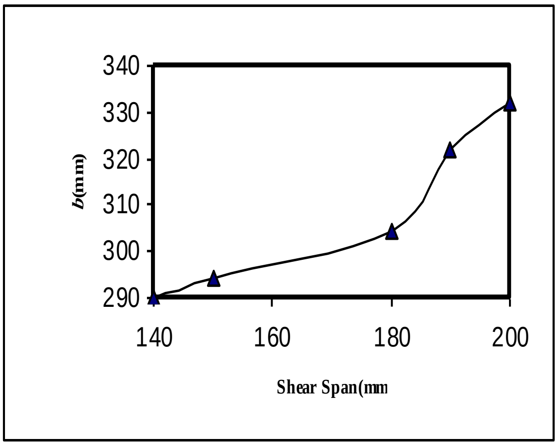

Fig. (9) Optimum corbel width vs. shear span

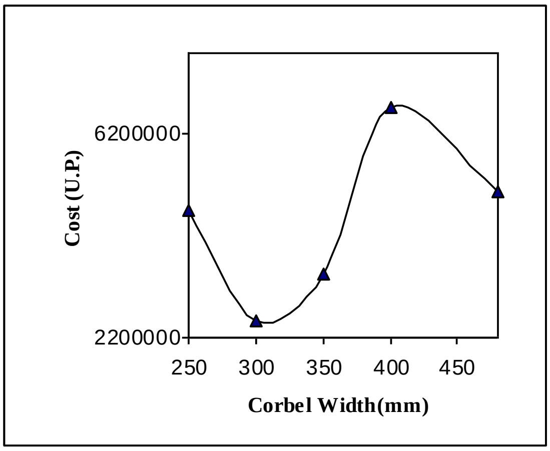

Fig. (10) Minimum total cost vs. corbel width

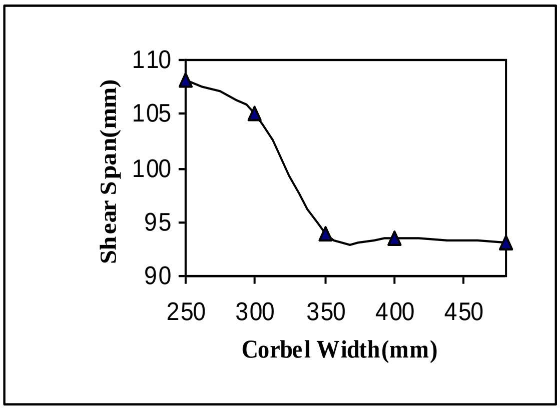

Fig. (11) Optimum shear span vs. corbel width

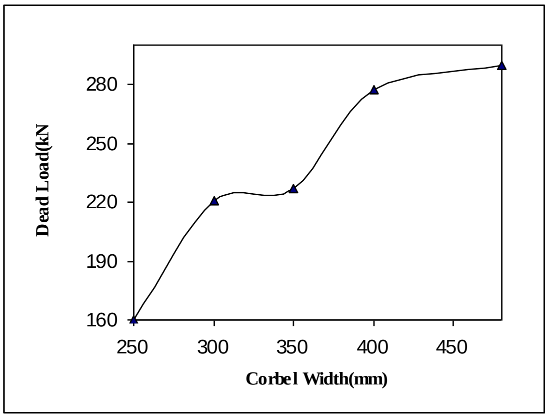

Fig. (12) Optimum dead load vs. corbel width

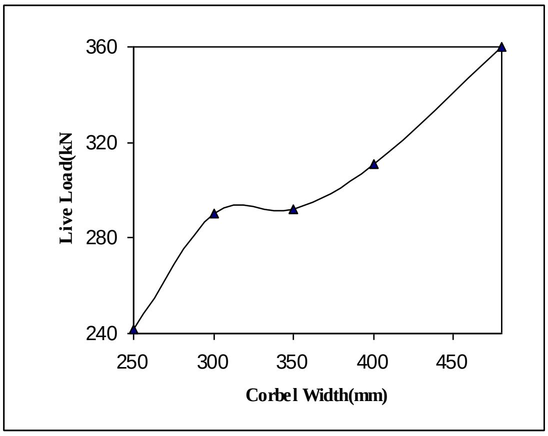

Fig. (13) Optimum live load vs. corbel width

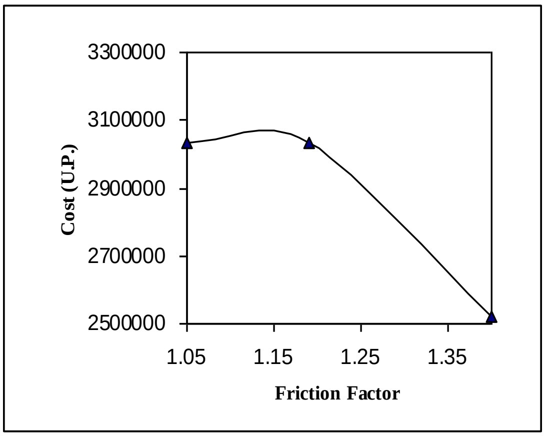

Fig. (14) Minimum total cost vs. friction factor

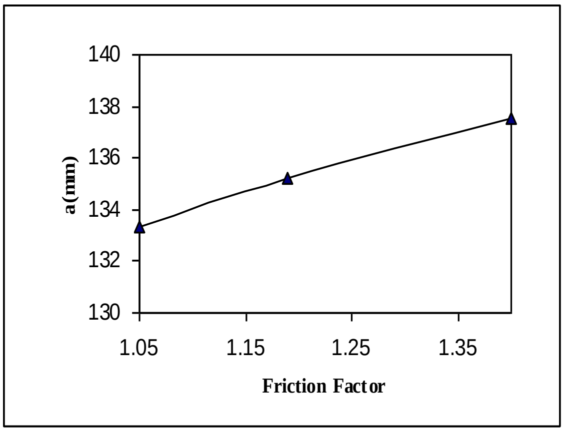

Fig. (15) Optimum shear span vs. friction factor

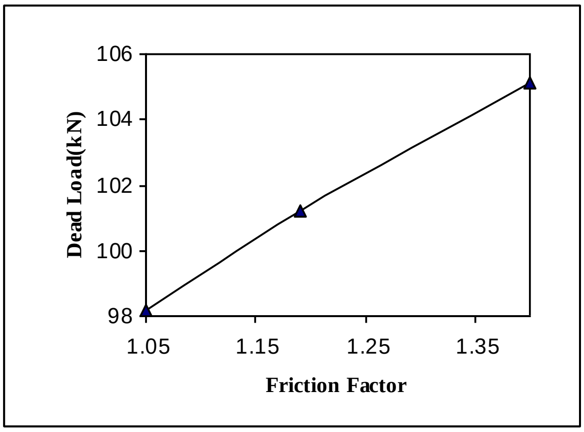

Fig. (16) Optimum dead load vs. friction factor

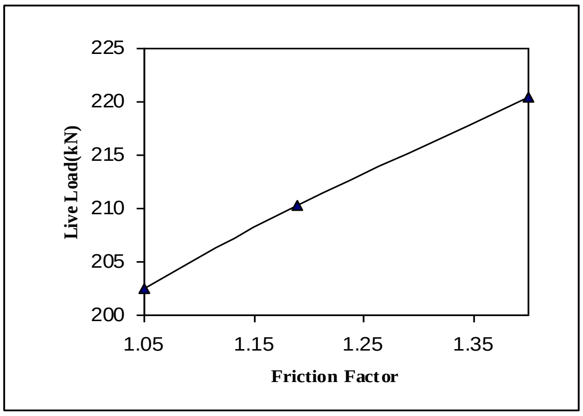

Fig. (17) Optimum live load vs. friction factor

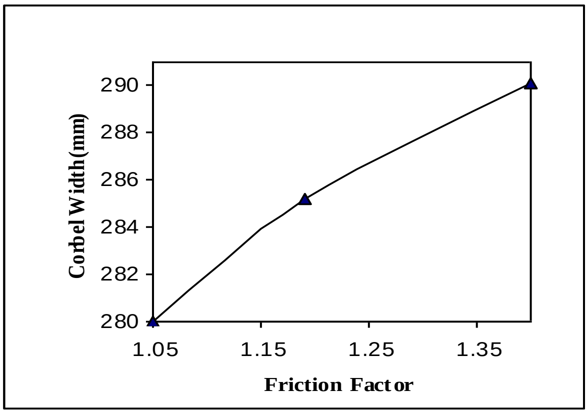

Fig. (18) Optimum corbel width vs. friction factor

التصميم الاقتصادي للأكتاف (CORBELS)

حسن جاسم محمد ألبدري

مدرس

قسم الهندسة المدنية – جامعة تكريت

الخلاصة

تمت دراسة تطبيق الطريقة المثلى على مسألة التصميم الإنشائي للأكتاف (CORBELS) ، باعتبار الكلفة الكلية للجسر كدالة هدف وبعض الخواص الهندسية (م44ثل فضاء القص والحمل الميت والحمل الحي وعرض الجسر) كمتغيرات تصميميه. تمت كتابة برنامج حاسبة لحل الأمثلة العددية بالاستناد إلى معادلات مواصفات المعهد الأمريكي للخرسانة ومتطلبات ومعايير التصاميم الخرسانية.

لقد برهن بان الكلفة الكلية للجسر تزداد بزيادة فضاء القص وتقل بنقصان معامل الاحتكاك للإنشاء المشترك.

الكلمات الدالة

أكتاف الجسر ، أمثلية عددية، تصميم