DYNAMIC BEHAVIOR OF PLATE HEAT EXCHANGER

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ABSTRACT

The mathematical modeling of plate heat exchanger was studied by introducing rectangular pulse change in inlet cold water flow rate and measuring the temperature of outlet cold water. A theoretical model for the plate heat exchanger was developed based on the heat balance and used the frequency response analysis techniques to determine the best model parameters. The results show that the dynamic behavior of plate heat exchanger can be approximated by a time delay with first order system and a closed fit between the theoretical coefficients of the system and these data by using frequency response method.

KEYWORDS

Plate Heat Exchanger, Frequency Analysis, Mathematical Model.

NOTATIONS

A_s: Surface area of heat exchanger. (m^2)

D_H: Hydraulic diameter of plate of heat exchanger. (m)

C_P: Specific heat capacity. (kj/kg. ^OC)

L: Length of the heat transfer path (m)

M: Mass of fluid in the element dx. (kg)

Q: Mass flow rate. (kg/hr)

T_{CI}, T_{CO} : Temperature of inlet and outlet cold water respectively. (^OC)

T_H: Temperature of hot water. (^OC)

T_X: Temperature of hot water at element dx. (^OC)

NOTATIONS-Continued

U: Overall heat transfer coefficient. ($W/m^2 . {}^{O}C$)

V: Velocity of water in the plate of heat exchanger. (m/sec)

τ: Time constant of the heat exchanger. (sec)

τ_D: Time delay constant of the heat exchanger. (sec)

INTRODUCTION

The performance of automatically controlled process plants depends on the dynamic interaction of all the components in the control loop. The dynamic characteristics of process components and of the overall plant indicated by frequency response data or by transient response data. From the control standpoint frequency response data are more useful. It is generally easier to compute the frequency response characteristics than the transient response characteristics since the frequency response is obtained directly from the transfer function of the process, while the transient response requires the taking of inverse Laplace transform [1].

The utility of mathematical models of chemical process is often limited by their complexity. Since exact analytical solutions to many important differential equations are unobtainable, the numerical calculations are often required for the comparison of theoretical prediction with observation. In the field of control system analysis, this difficulty has been circumvented by the employment of approximate models of physical system dynamics. Model parameters are estimated by sinusoidal or pulse testing techniques which require only the Fourier transform of the differential equation approximating the system.

This paper presents a study of the dynamic characteristics of plate heat exchangers. Although this type problem has been treated in general terms by Roetzel[2], Lakshmann[3], Kahn[4], sharifi[5], and others. Das and

Roetzel[6] solved the equations for a plate heat exchanger with phase lag in the fluid entry temperatures. This problem arises because of the time taken to transit the headering system. At fluid exits from the exchanger core, they assumed zero temperature gradients.

In describing the experimental investigation of the dynamic behavior, the frequency response technique was chosen, as result obtained by this method is usually highly reproducible and is relatively easy to compare with prediction from theoretical models. This technique was applied in a number of dynamic studies of chemical process equipment. Including stirred tank reactor, heat exchanger, and spray drier, turbulent flow system, bubble caps plates system and packed beds. This technique has been treated by Gangwall and Silveston [7], Wakao and Tanisho [8], Al-Dawery [9[, and Lokman[10].

The advantages of frequency response methods for the dynamic analysis of physical systems are well known. Some of the most fruitful techniques for process control and analysis are based on frequency response methods. One especially interesting use is the comparison of experimental frequency response data for a system with the response calculated for a postulated model, to obtain a quantitative estimate of the range of validity of the model in describing the system.

This paper presents a study of the dynamic characteristic of plate heat exchanger by introducing rectangular pulse change in inlet flow rate of the cold water and use the frequency response analysis technique to evaluate the system parameters.

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THEORY

1-Mathematical Model

A simplified representation of the plate heat exchanger is shown in Fig.(1). The process fluid (cold water) flows through the side of the frame is heated by hot water flows at another side of the frame. The differential energy balance for the process fluid over the volume element of length (dx) at any time is given by:

Heat flow in – heat flow out + heat transferred = heat accumulation in the element (dx)

$$QC_P T_X(x) - QC_P T_{X+\Delta X}(x) + UA_S(T_H(t) - T_X) = MC_P \frac{\partial T_X}{\partial t} \qquad \text{.......... (1)}$$

$$MC_P \frac{\partial T_X}{\partial t} + UA_S T_X(t) = UA_S T_H(t) - QC_P \frac{\partial T_X}{\partial x} \Delta X \qquad \text{........... (2)}$$

where:

$$A_S = \pi D_H \Delta X \,,\; M = (\frac{\pi}{4} D_H{}^2 \Delta X)\rho \,,\;\; Q = \frac{\pi}{4} D_H{}^2 V\rho$$

$$\frac{D_H \rho C_P}{4} \frac{\partial T_X}{\partial t} + U\,T_X(t) = U\,T_H(t) - \frac{D_H \rho V C_P}{4} \frac{\partial T_X}{\partial x} \qquad \text{........... (3)}$$

$$\tau \frac{\partial T_X}{\partial t} + T_X(t) = T_H(t) - \tau V \frac{\partial T_X}{\partial x} \qquad \text{.............(4)}$$

$$Where\;\; \tau = \frac{D_H \rho C_P}{4U}$$

Assuming zero initial conditions, taking Laplace transformation of Eq.(4)

$$\tau s T_X(s) + T_X(s) = T_H(s) - \tau V \frac{dT_X}{dX} \qquad \text{............. (5)}$$

$$\ln\frac{(\tau s+1)T_{CO} - T_H(s)}{(\tau s+1)T_{CI} - T_H(s)} = \frac{-L}{\tau V}(\tau s+1) = -\tau_D s - \frac{L}{\tau V} \qquad \text{.............(6)}$$

$$\frac{(\tau s+1)T_{CO}-T_H(s)}{(\tau s+1)T_{CI}-T_H(s)} = \exp(-\tau_D s)\ \exp(-\frac{L}{\tau V}) \qquad \text{……….(7)}$$

$$where\ \tau_D = \frac{L}{V}$$

If T_H is constant all times, $T_H(s) = 0$

$$G(s) = \frac{T_{CO}(s)}{T_{CI}(s)} = \exp(-\tau_D s)\ \exp(-\frac{L}{\tau V}) \qquad \text{……….. (8)}$$

If T_{CI} is constant, $T_{CI}(s)=0$

$$G(s) = \frac{T_{CO}(s)}{T_H(s)} = \left(\frac{1}{\tau s+1}\right)(1-\exp(-\tau_D s)\ \exp(-\frac{L}{\tau V}) \qquad \text{…………(9)}$$

2- Frequency Analysis

One of the most useful and practical methods for obtaining experimental dynamic data from many chemical engineering processes is pulse testing. It yields reasonably accurate frequency-response curves and requires only a fraction of the time that directs sine-wave testing takes.

An input pulse m(t) of fairly arbitrary shape is put into the process. This pulse starts and at the same value and is often just a square pulse (i.e., a step up at time zero and a step back to the original value at a later time t_m).

The input and output functions are then Fourier-transformed which then be used divided to give the system transfer function in the form of the frequency domain G(iω). The details of one procedure for accomplishing this Fourier transformation are discussed in the following sections, and a little digital computer program. Alternative methods include the use of "Fast Fourier Transforms" which are available in most computing centers.

In theory only one pulse input is required to generate the entire frequency response curve. In practice several pulses are usually needed to establish the required size and duration of the input pulse. Consider a process with an input m(t) and an output x(t). By definition, the transformer function of the process is:

$$G(S)=\frac{X(s)}{M(s)}=\frac{\int_0^\infty x(t)e^{-st}\,dt}{\int_0^\infty m(t)e^{-st}\,dt} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

We now go into the frequency domain by substituting s = iω.

$$G(i\omega)=\frac{X(i\omega)}{M(i\omega)}=\frac{\int_0^\infty x(t)e^{-i\omega t}\,dt}{\int_0^\infty m(t)e^{-i\omega t}\,dt} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The numerator is the Fourier transformation of the time function x(t). The denominator is the Fourier transformation of the time function m(t). Therefore the frequency response of the system G(iω) can be calculated from the experimental pulse test data x(t) and m(t).

$$G(i\omega)=\frac{\int_0^\infty x(t)\cos(\omega\, t)dt-i\int_0^\infty x(t)\sin(\omega\, t)dt}{\int_0^\infty m(t)\cos(\omega\, t)dt-i\int_0^\infty m(t)\sin(\omega\, t)dt} \qquad \ldots\ldots\ldots\ldots\ldots (12)$$

$$G(i\omega)=\frac{A-iB}{C-iD}=\frac{(AC+BD)+i(AD-BC)}{C^2+D^2} \qquad \ldots\ldots\ldots\ldots\ldots (13)$$

where

$$A = \int_0^{tx} x(t)\cos(\omega t)\,dt \qquad B = \int_0^{tx} x(t)\sin(\omega\ t)\,dt$$

$$C = \int_0^{tm} m(t)\cos(\omega t)\,dt \qquad D = \int_0^{tm} m(t)\sin(\omega\ t)\,dt$$

$$G(i\omega) = \mathrm{Re}\big[G(i\omega)\big] + i\,\mathrm{Im}\big[G(i\omega)\big] \qquad \text{………….. (15)}$$

From Equation (11), we wish to evaluate the Fourier integral transform (FIT) of x(t).

$$FIT \equiv \int_0^{\infty} x(t)e^{-\omega t}\,dt \qquad \text{…………...(16)}$$

We can break up the total interval (0 to t_x) into a number of unequal subintervals of length Δt_k. Then the FIT can be written, with no loss of rigor, as a sum of integrals:

$$FIT \equiv \sum_{k=1}^{N}\left(\int_{t_{k-1}}^{t_k} x(t)e^{-i\omega t}\,dt\right) \qquad \text{……………(17)}$$

$$FIT \approx \sum_{k=1}^{N}\left(\int_{t_{k-1}}^{t_k}\big[\alpha_{0k} + \alpha_{1k}(t - t_{k-1})\big]e^{-i\omega t}\,dt\right) \qquad \text{…………….(18)}$$

$$FIT = \sum_{k=1}^{N} I_k \qquad \text{…………….(19)}$$

Each of the I_k integrals above can be evaluated analytically.

Finally, the Fourier transformation of x(t) becomes:

$$I_k = \int_{t_{k-1}}^{t_k}\big[\alpha_{0k} + \alpha_{1k}(t - t_{k-1})\big]e^{-i\omega t}\,dt$$

$$= \left[\frac{-\alpha_{0k}}{i\omega}e^{-i\omega t}\right]_{t_{k-1}}^{t_k} + \int (t - t_{k-1})\ e^{-i\omega t}\,dt \qquad \text{…………..(20)}$$

Equation (20) looks a little complicated but it is easily programmed on a digital computer. Appendix A gives a FORTRAN program that reads input and output time functions from a file, calculates the Fourier transformations of the input and of the output, divides the two to get the transfer function G(iω), and prints out log modulus and phase angle at different values of frequency. After obtaining the log modulus and phase angle, the Bode diagram can be plotted from which the system and transfer function can be obtained. This can represented the system model.

The steady state gain of the transfer function is G(0) or just the ratio of the areas under the input and output curves.

$$K_P = G(0) = \frac{\int_0^{t_x} x(t)\,dt}{\int_0^{t_m} m(t)\,dt} \qquad \ldots\ldots\ldots\ldots\ldots \quad (21)$$

If the input pulse is a rectangular pulse of height h and duration D, it Fourier transformation is simply:

$$\int_0^{\infty} m(t) e^{-i\omega t}\,dt = h\int_0^{D} e^{-i\omega t}\,dt = -\frac{h}{i\omega}\left[e^{-i\omega t}\right]_{t=0}^{t=D}$$

$$= \frac{h}{i\omega}\left(1 - e^{-i\omega D}\right) \qquad \ldots\ldots\ldots\ldots(22)$$

EXPERIMENTAL WORK

1- Description of The Experimental Equipment

A Laboratory plate heat exchanger control system is consisting of heat exchanger, heating tank and control devices. The system is show in Fig.(1). The type of heat exchanger is plate; the surface area is 0.6 m^2 and is supplied two inlets streams-heating and process fluid- with four temperature measurements for inlet and outlet streams. The capacity of heating tank is 0.05 m^3 is supplied with electrical heater to heat the water. Hot water supplied to the heat exchanger at pressure 3 barg and with a maximum flow rate of a 30 lit./min and the process fluid supplied from tap water. The two rotameter having stainless steel float with range of flow (1–20 lit./hr) of water at about 20°C each were employed for measuring the flow rate of the inlet streams. The control system consists of the PID controller, control valve, temperature transmitter, I/P converter and air filter with regulator.

2-Experimental Arrangement

The runs were carried out for the heat exchanger, the equipment was first prepared as follows: -

1. Heating tank was supplied with water and electrical supply is connected to pump, heater and the control devices of temperature.
2. Adjust the air supply to control instruments using the regulator to give a pressure of 21± 1 psig.
3. Select switch is in position valve and transfer switch is in position manual.
4. Open fully valve V2, V3 and keeping V4 closed.
5. Adjust process (cold) fluid flow rate to 4 lit/min by adjusting V1 and adjust hot fluid flow rate to 4 lit/min by adjusting V4 and wait for steady state.
6. Record heating and process fluids inlet and outlet temperatures, T1, T2, T3 and T4.
7. Rectangular pulse change of the process fluid at 5 lit./min. with duration equal to 120 sec. and record heating and process fluids inlet and outlet temperatures, T1, T2, T3 and T4.

8. Repeat the step (7) with change flow rate of process fluid from 4 to 3 and 6 lit./min. with duration equal to 120 sec. and change the duration of 100 and 140 sec. at change of the flow rate of 5 lit./min.

RESULTS AND DISCUSSION

The dynamic response of the plate heat exchanger by rectangular pulse change in flow rate of the cold water is shown in Fig.(2) and Fig.(3). This behavior can be approximated by a system with a 4 seconds dead time component with first order system. The experimental time constant is equal to 1.25 second and time delay 4 seconds by using frequency response method while the theoretical experimental time constant is equal to 2.6 seconds and time delay 5 seconds.

Fig.(4) to Fig.(9) show the frequency response characteristics of the heat exchanger plotted on Bode type diagram. The phase lag increase from to zero to (-180) with increasing frequency, and amplitude ratio decrease with increasing frequency. The lumped approximation for magnitude ratio fits the data well. On the other hand the theoretically computed curve lies higher than the data. Part of this difference is due to the fact that the experimental data include the response of the temperature measuring and the dynamic behavior of the insulation of the heat exchanger while the theoretical curve indicates the response of the heat exchanger alone.

The curves showing the phase lag angle indicate that the experimental results are not in good agreement with the theoretical results. The theoretical curve lies above the experimental data, and the data indicate a more rapid fall-off than the theoretical curve. In addition to the sources of deviation in the magnitude ratio there was a distance velocity lag between the heat exchanger and the thermocouple measuring the outlet temperature. Thus it is to be expected that the lag angle would be greater than that indicated from the response of the heat exchanger alone.

The frequency domain modeling is the facility with which analytical frequency solutions can be obtained as compared with time solutions. However, the analytical inversion of the frequency solution to find the time solution is formidable. The lumped parameter model there is no change the form of the frequency solution as the values of the parameters change. The region of interest over which it is desired to fit a mathematical model is quite often conveniently expressed in the frequency domain.

Equations for the dynamic characteristics of a simple distributed system have been derived. The frequency response characteristics have been computed and it has been shown that distributed periodic forcing leads to resonance in magnitude ratio and phase lag angle.

CONCLUSIONS

Mathematical modeling is often thought of as being concerned with time domain signals. However, for many problems it is much simpler and computationally advantageous to evaluate the model in the frequency domain. Another distinct advantage of frequency domain regression is that frequency domain specifications and restrictions can be easily incorporated. Frequency domain model evaluation allows one to take this into consideration; this may involve the elimination of a high or low frequency range or the rejection of a band of frequencies that perhaps contains noise or a spurious frequency component.

The results show that the dynamic behavior of plate heat exchanger can be approximated by a time delay with first order system(see equation (9)) and a closed fit between the theoretical coefficients of the system and these data by using frequency response.

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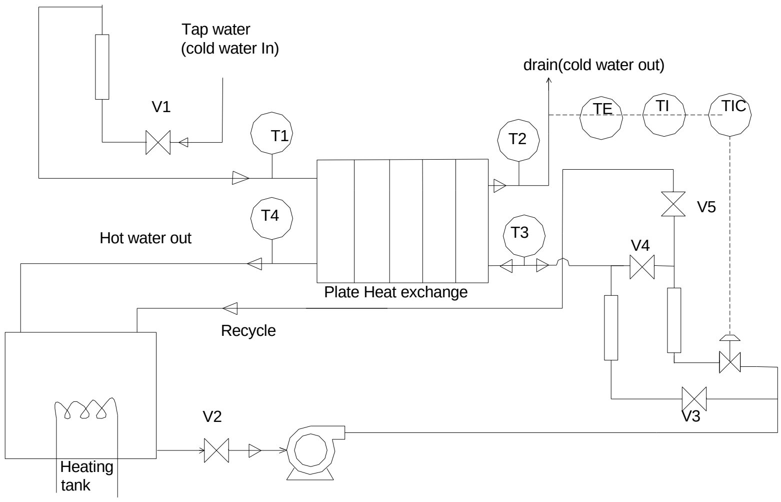

Figure (1) Schematic diagram of experimental plate heat exchanger.

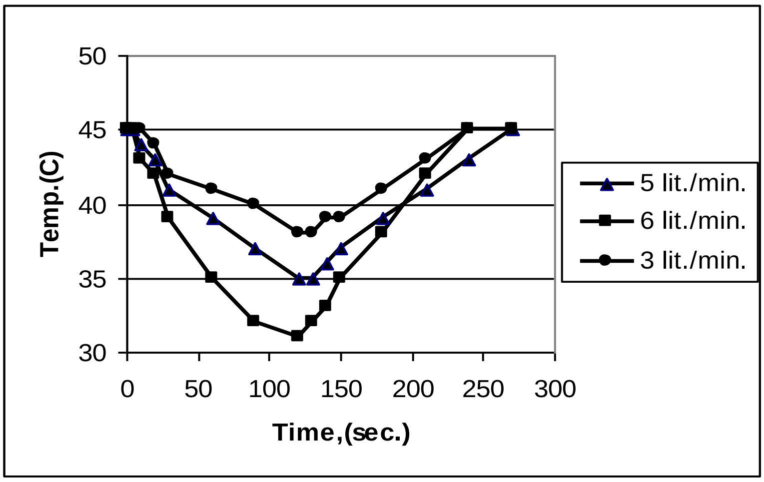

Figure (2) Experimental response of process fluid temperature for heat exchanger to rectangular pulse change with different flow rate.

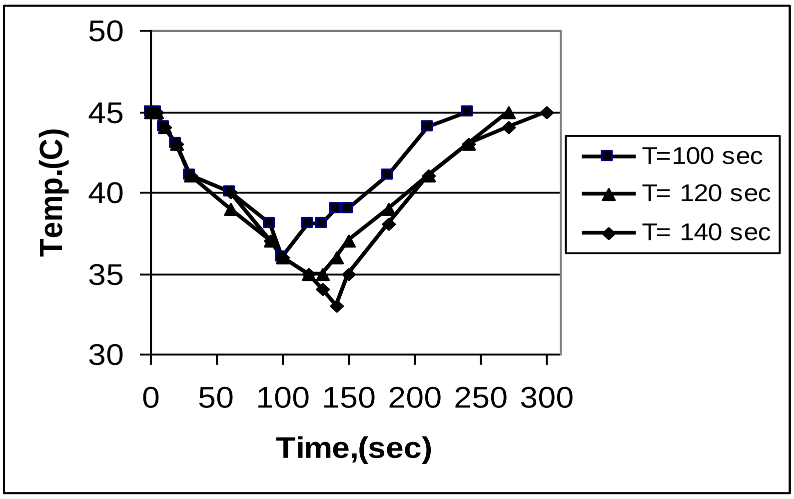

Figure (3) Experimental response of process fluid temperature for heat exchanger to rectangular pulse change with different duration.

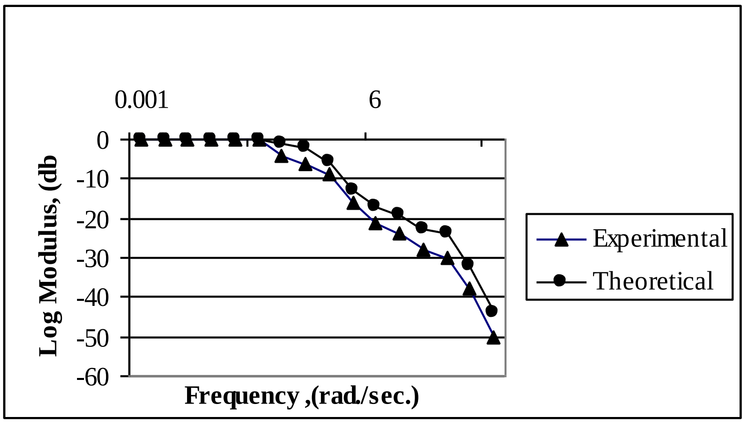

Figure (4) Bode plot of heat exchanger (log modulus) at flow rate 5 lit./min. and duration equal to 120 sec.

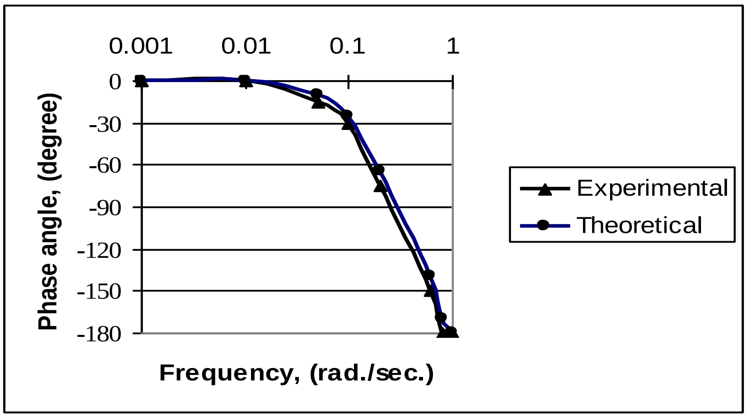

Figure (5) Bode plot of heat exchanger (phase angle) at flow rate 5 lit./min. and duration equal to 120 sec.

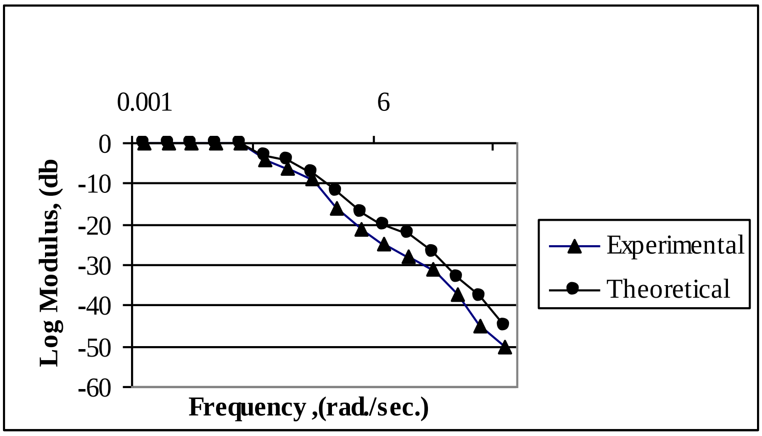

Figure (6) Bode plot of heat exchanger (log modulus) at flow rate 6 lit./min. and duration equal to 120 sec.

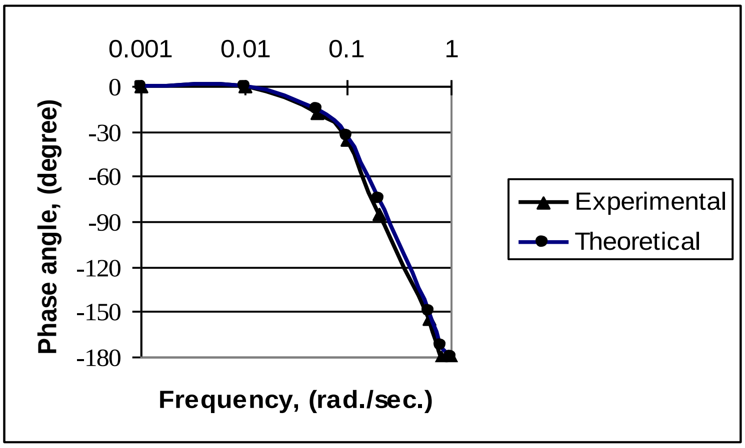

Figure (7) Bode plot of heat exchanger (phase angle) at flow rate 6 lit./min. and duration equal to 120 sec.

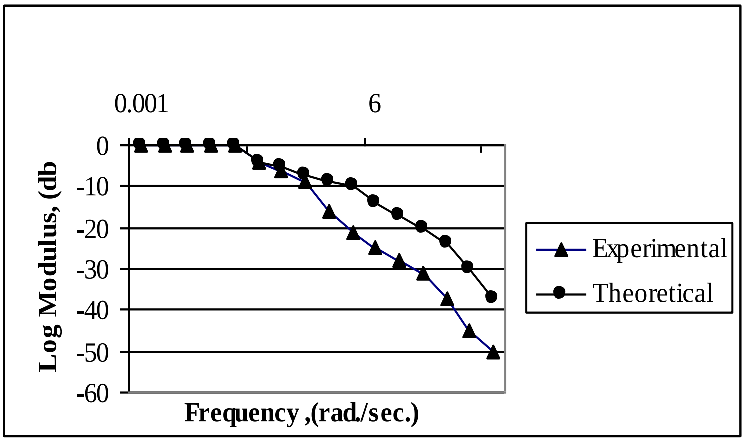

Figure (8) Bode plot of heat exchanger (log modulus) at flow rate 5 lit./min. and duration equal to 100 sec.

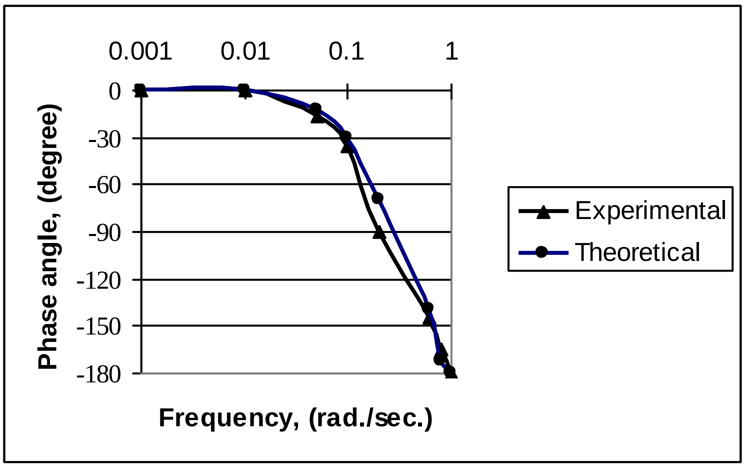

Figure (9) Bode plot of heat exchanger (phase angle) at flow rate 5 lit./min. and duration equal to 100 sec.

السلوك الديناميكي للمبادل الحراري ذو الصفائح

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مدرس

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الخلاصة

تم دراسة التمثيل الرياضي للمبادل الحراري ذو الصفائح من خلال تعريضه لتغيير من نوع نبضة مستطيلة بمعدل الجريان الماء البارد الداخل إلى النظام وقياس درجة الحرارة الماء البارد الخارج من المبادل الحراري . تناول البحث اشتقاق الموديل النظري للمبادل الحراري ذو الصفائح بالاعتماد على الموازنة الحرارية وتم استخدام تحليل الاستجابة الترددية لحساب أفضل المعاملات للموديل الرياضي. أظهرت النتائج من خلال دراسة السلوك الديناميكي للمبادل الحراري باعتباره نظام درجة أولى مضافا إليه عنصر التعوق الزمني وتقارب بين معاملات النظام النظرية و معاملات النظام المستخرجة بواسطة طريقة الاستجابة الترددية.

الكلمات الدالة

المبادل الحراري ذو الصفائح, الأستجابة الترددية, التمثيل الرياضي.