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ECONOMICAL DESIGN OF CIRCULAR FOOTINGS ADJACENT TO SLOPES ON SANDY SOILS

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ABSTRACT

The analysis presented here introduces three optimization techniques namely, Hooke and Jeeves, Fletcher-Reeves and Davidon-Fletcher-Powell as applied to design of the circular footing adjacent to slopes. A computer program was developed to solve this design problem using the conventional structural design approach in conjunction with these methods, A simple study was performed to detect the sensitivity of the objective function to its design variables. A further parametric study was performed regarding the geometric configurations of the footing and loading conditions in order to provide the geotechnical engineer with some useful design curves. Hooke and Jeeves method has been proved to be very instructive in exposing the effect of the other methods.

It has been proved that the minimum cost of the circular footing increases with the increase of the load whereas it decreases as the angle of internal friction increases and the D_{cl}/B ratio (column diameter/diameter of footing).

Key words: optimization, conventional design, slope, circular footing, sandy soil.

NOTATIONS

As	Area of steel
B	Footing diameter
b	Ground projection (the distance from the edge of slope to the footing)
Dc	diameter of column
c	Cohesion of base soil
cl	Concrete cover
Con	Cost of concrete
Cex	Cost of excavation
Cc	Cost of backfilling works
Cst	Cost of reinforcing steel
d	Footing effective depth
db	Diameter of steel bar
D_f	Embedment depth of footing
DFP	13avidon- Fletcher- Powell method
Dw	Maximum required depth for wide beam action
dc,dq,dγ	Depth factors for the Hansen's bearing capacity equation
	Stress- Strain modulus of soil
E_s	Stress- Strain modulus of footing
ESS	Fletcher-Reeves Method

FR	Objective function
F()	Compressive strength of concrete
fc'	Number of function evaluations
FE	Yield strength of steel
fy	Hooke & Jeeves method
HJ	Step length
HZ	Influence factor which used in
I_I	settlement computations
	Modulus of subgrade reaction
Ks	Effective length beneath column
L1	Required length of reinforcing steel
Lb	Load factor
LF	Bending moment
M	Equivalent term
m	Bearing capacity factors forr the
Nc',Nq'	Hansen's bearing capacity equation
	corrected for slope
	Bearing capacity factors for the
Nγ	Hansen's bearing capacity equation
P	Working applied load on column
Pc	Price of concrete
P_{ex}	Price of excavations
P_f	Price of backfilling works
P_{st}	Price of reinforcing steel
$\overline{q}$	Effective overburden pressure at base level
q	Ultimate applied pressure at the left end of the footing
q_{al}	Allowable soil pressure
q_{max}	The maximum applied pressure

q_o	Intensity of contact pressure
q_u	Hansen's ultimate bearing capacity of base soil
R	Resultant force of the applied loads on the footing
r^{γ}	Reduction factor for limited influence of base width
S_c,S_q,S_γ	Shape factors for the Hansen's bearing capacity equation
Scd	Maximum deformation beneath the footing base
Sd	Differential settlement beneath the footing base
SF	Safety factor against bearing capacity failure
Si	Maximum immediate settlement
S_L	The slope of the pressure line
T	Footing thickness
TC	Total thickness of soil layer
TH	Thickness of soil layer beneath footing base
UR	Ultimate ratio
V	Actual shear force
Vst	Volume of reinforcing steel
w	Half width of column
X	Design vector
x	Design variable
Z	Total cost of the circular footing
γ	Unit weight of the base soil
γc	Unit weight of the concrete
ϕ	Angle of internal friction of soil

ρ	Reinforcement ratio
ρ	Unit mass of steel.
υ	Poisson's ratio

INTRODUCTION

It is evident that, for any engineering design problem, engineers have to take many decisions at several stages to either minimize the effort required or maximize the desired benefit. This decision-making problem can be rectified through the use of available facilities in the field of "Operations Research" to help the designer in choosing the appropriate criterion to achieve the best results satisfying design restrictions[1]. Mathematical programming techniques are generally studied as a part of operations research [2].

PURPOSE OF STUDY

The principal purpose of this research is to investigate the usefulness of optimization methods to search for the most economical design of circular footing adjacent to slopes and to detect the sensitivity of the objective function to its design variables in order to achieve a safe and economical design.

FORMULATION OF THE PROBLEM

A formula representing the total cost of the footing (COST) was considered as the objective function. Figure (1) shows four independent design variables were selected namely; footing diameter (B), thickness of the footing (T), embedment depth (DF) and ground projection (b). Soil properties were treated as constant quantities.

PROGRAMME USER'S MANUAL

The program for the design of circular footing has been written in "QUICK-BASIC", Optimization program carrying out the minimization process were defined as main program with termination accuracy of the step length less than 1.0 E-X, A subroutine was linked to the main program. It contains the necessary computations for structural analysis using the conventional approach.

OBJECTIVE FUNCTION

The total cost of the circular footing was considered as the objective function, It can be calculated as follows:

$$\text{Cost (U.P.)} = C_{con} + C_{ex} + C_{f} + C_{st} \quad .....(1)$$

Where:

Cost (U.P.) = total cost (unit price).

C_{con} = cost of concrete (unit price).

C_{ex} = cost of excavations (unit price).

C_f = cost of backfilling works (unit price).

C_{st} = cost of steel reinforcement (unit price).

A. Cost of Concrete

C_{con} = Vol. of Concrete * P_c

$= \pi/4 * B^2 * T * Pc$ ………(2)

Where:

B = footing diameter (m).

T = footing thickness (m).

P_c = price of concrete including materials and labour (unit price per cubic meter).

B. Cost of Excavation Works

$C_{ex} = \pi/4 * B^2 * DF * P_{ex}$ ……....(3)

Where:

DF = embedment depth of footing (m).

P_{ex} = price of excavation works and labour (unit price per cubic meter).

C. Cost of Backfilling works

$C_f = \pi/4 * B^2 * (DF-T) * P_f$ ……....(4)

Where:

P_f = price of backfilling works, materials and labour (unit price per cubic meter).

D. Cost of Reinforcing Steel

C_{st} = Vol. of steel * density * P_{st}

$= V_{st} . \rho s . P_{st}$ ……………....(5)

Where:

V_{st}=total volume of reinforcing steel (m^3)

ρ_s = unit weight of steel (ton/m^3)

P_{st} = price of steel including materials & labour (unit price per ton) .

CONSTRAINTS

In this research two main types of constraint were considered; the geotechnical and structural constraints .Each type is discussed for the circular footing problem in the following sections;

A. Geotechnical Constraints

1. Stability against base failure

i.) The maximum applied pressure under-the footing base (q_{max}) should not exceed the allowable bearing capacity (q_{all}),

$$\frac{q_u}{q_{max}} \geq SF \qquad \text{…………………..(6)}$$

Where:

q_{max} = The maximum applied pressure (kN/m^2) ,

$$= \frac{P}{\frac{\pi}{4} B^2} \qquad \text{…...(6a)}$$

P = working applied loads on column (kN).

q_u = Hansen's ultimate bearing capacity of base soil (kN/m^2).

$$= cN'_c s_c d_c + qN'_q s_q d_q + 0.5 B N_\gamma s_\gamma d_\gamma r_\gamma \text{ ref.[3]}$$

…………….……..(6b)

c = the cohesion of the base soil (kN/m^2)

q = effective overburden pressure at footing base level (kN/m^2)

$=\gamma.D_f$ ………………..(6c)

γ = unit weight of the base soil (kN/m^2).

N'_c, N'_q, N'_γ = bearing capacity factors for the Hansen's bearing capacity Equation which depends on ϕ only.

N'_q *and* N'_c = corrected bearing capacity factors for the Hansen's bearing capacity equation.

$$N_\gamma = 1.5(N_q - 1)\tan\phi \quad \text{…………….(6d)}$$

ϕ = angle of internal friction of the base soil (degrees).

s_c, s_q, s_γ = shape factors for the Hansen's bearing capacity equation, and for circular footing are:

$$\left.\begin{aligned} S_c &= 1 + \frac{N_q}{N_c} \\ S_q &= 1 + \tan\phi \\ S_\gamma &= 1 - 0.4 \end{aligned}\right\} \text{……………………….(6e)}$$

d_c, d_q, d_γ = depth factors for the Hansen's bearing capacity equation,

$$\left.\begin{aligned} d_c &= 1 + 0.4K_1 \\ S_q &= 1 + 2\tan\phi(1-\sin\phi)^2.K_1 \\ S_\gamma &= 1 \end{aligned}\right\} \text{……(6f)}$$

$$\left.\begin{aligned} K_1 &= D_f / B \; when \; D_f / B \le 1 \\ K_1 &= \tan^{-1}(D_f / B)(radians) \; when \; D_f / B > 1 \end{aligned}\right\}$$

…..…………………….(6g)

R_γ = reduction factor for limited influence of footing width,

$$\left.\begin{aligned} &= 1.0 \; for \; B \le 2m \\ &= 1 - 0.25\log(B/2) \; for \; B > 2m \end{aligned}\right\} \text{…..(6h)}$$

SF = reduction factor against bearing capacity failure. =2

2. Footing settlement

The maximum immediate settlement (S_i) and differential settlement (S_d) must be within the allowable limits [3].

$S_i \le$ 3.81 cm (1.5 in) ..…………….(7)

$S_d \le$ 2.54 cm (1 in) ……………...(8)

3.Protectoin Against Environmental Effects

The footing should be constructed below the zone of seasonal volume changes. Thus, the following constraint will be introduced:

2m $\ge$ DF $\ge$ 0.9m …………(9)

B. Structural Constraints

1. Shear failure

i.) Wide-beam shear

The maximum shear stress due to wide-beam shear $(v_c)_w$ must be within concrete strength [4].

$$(v_c)_w = 0.17 \times 0.85 \times \sqrt{f'_c} \quad \text{...………(10)}$$

ii.) Punching shear

The maximum sheer stress due to punching shear (diagonal tension) $(v_u)_p$ must be within the concrete strength see Fig.(2) [5].

$(v_u)_p \leq 0.33 \times 0.85 \times \sqrt{f'_c}$ ....(11)

2. Reinforcement Ratio for Bending Moment

The reinforcing ratio for bending moment at any section should not be less than(ρ_{min}) and it should not be more than (ρ_{max}) [6].

$\rho_{\min} \leq \rho_i \leq \rho_{\max}$ ……………………..(12)

Where :

ρ_i = reinforcement ratio for bending moment at any section.

ρ_{min} = minimum reinforcement ratio,

$$= \frac{1.4}{f_y} \quad (for\ beams) \quad …………(12a)$$

f_y = yield strength of steel (MPa)

ρ_{max} = maximum reinforcement ratio,

$$= 0.75 \frac{0.85 f'_c}{F_y} \beta_1 \frac{600}{600 + F_y} ……….(12b)$$

$$\left.\begin{aligned} \beta_1 &= 0.85\ when\ f'_c \leq 28 N/mm^2 \\ &= 0.85 - 0.0275 (f'_c - 28)\ when\ f'_c > 28 N/mm^2 \end{aligned}\right\}$$

..…………..(12c)

C- Dimension Constraints

The footing diameter (B), footing depth of embedment (D_f) and the footing thickness (T) are governed by practical considerations.

$B \geq 3D$ ………………………...(13)

$2m \geq DF \geq \max (T, 0$………………(14)

$T \geq 0.25m$ ……………..(15)

It should be noted that, there is no need for an upper limit for footing thickness since any large value of (T) will be discarded in favour of cost minimization. Hence, the optimization problem can be stated as:

Find $X= [\ B\ T\ DF\ b]^T$ that minimizes Eq. (1) subject to the constrains defined by Equations (4) to (15). The problem of a circular footing design can be solved as an unconstrained minimization problem by giving the cost function a high value upon violation of any constraint in order to discard the point (i.e., values of design variables) generated this situation.

NUMERICAL EXAMPLE

This numerical example illustrates the application of the used optimization methods to the circular footing design problem and confirming their utility to reach the optimum solution., for more details, the reader is referred to [2,7]. The following values were assigned to the input parameters of the subroutine "CON ". [2]

P = 875 kN	LF = 1.6
w = 0.15 m	ϕ = 30 Deg.
c = 0.0 kN/m^2	υ = 0.3

E_s = stress - strain modulus of soil (kN /

m^2) = Ks. B $(1-\upsilon^2) I_s \cdot I_f$

Ks = modulus of subgrade reaction (kN / m^3) $I_s = I_1 + \left(\frac{1-2\upsilon}{1-\upsilon}\right) I_2$ ……..(16)

I_i = influence factors which depend on (L/B), thickness of stratum, Poisson's ratio (υ) and embedment depth (D_f),

If = 1 γ_{soil} = 17 kN / m^3

γ_{con} = 24 kN / m^3 f'_c = 21 MPa

F_y = 375 MPA ρ_s =78.5 kN/m^3

P_c = 325 (unit price per ton)

P_{st} = 770 (unit price per cubic meter)

P_{ex} = 7 (unit price per cubic meter)

P_f = 8 (unit price per cubic meter)

The above sample problem was solved by using three optimization methods and using three initial trial points, The following are the required input data for each one,

N = number of design variables =6, Hz - step length= 0.05

x(l) = B, X(2) = T, X(3) = DF, X(4) = b

The first initial trial values:

X(1)=3, X(2)= 1.0, X(3)=1.0, X(4)=1.0

The second initial trial values:

X(1)=2. 75, X(2)= 0.9, X(3) = 0.75, X(4)=0.75,

The third initial trial values:

X(1)=2.5, X(2)= 0.5, X(3)=0.9, X(4)=0.5

The results obtained are shown in Tables(l) and (2). Figures (3 and 4) show the convergence rate towards the minimum cost design of the circular footing adjacent to slope.

SENSITIVITY TO THE DESIGN VARIABLES

In order to specify the first order parameter among the design variables, a simple study was performed on the cost function via changing the values of the design variables one at a time. It can be deduced from Figs. (5) through (8) that, the cost of footing is more sensitive to the changes in the values footing width and thickness. The results demonstrate the minor effect of footing depth of embedment, DF and ground projection ,b as shown in Table(2).

PARAMETRIC STUDY

A parametric study was carried out regarding loading conditions. Angle of internal friction and column diameter to footing diameter ratio. The results are shown in Tables (3)through(5).

DISCUSSION

It can be observed from Tables (1) and (2) and from Figs.(3) and (4) that, Hooke and Jeeves method through the third trial point was more efficient in locating the minimum cost than the other optimization methods.

It is evident from Table (2) and Figs.(5) through (S) that the minimum cost is more sensitive to the changes in the footing width and thickness of the footing compared to the variations in footing embedment depth and ground projection.

It can be deduced from Fig. (9) that the slope angle has no effect on the minimum cost. Table (3) and Fig. (10) show that the minimum cost decrease as the friction angle increase upto ϕ=26, after that remains constant.

It is clear from Fig. (ll) and Fig. (l2) that, the minimum footing width and thickness decreases with the decrease of friction angle then they unchanged after the value $\phi = 26^o$. It can be observed from Table (4) and Fig. (13) that, the minimum cost increases as the load increases. This increase in the minimum cost is due to the increase in the optimum footing width and thickness as shown in Fig.(l4) and Fig.(l5).

Table (5) and Fig. (l6) demonstrate the significant effect of the column dia. to footing dia. ratio as obvious from Fig.(l7) and Fig.(l8).

CONCLUSIONS

1. The achievement of an economical foundation design can be handled as a problem of mathematical programming.
2. Optimization technique was successfully applied to the problem of circular footing design adjacent to slope on sandy soil.
3. The optimum cost of footing was more sensitive to the changes in the values of footing width and thickness.
4. The minimum cost was more sensitive to the changes in load ratio and internal friction angle than to the changes in column dia. to footing dia. ratio.
5. The slope near the footing was not effect on the minimum cost of footing.

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Table (1) The Results of Analysis for Optimization Methods

Variables	Hooke & Jeeves	FR	DFP
B (m)	2.12	2.25	2.312
T (m)	0.42	0.499	0.499
DF (m)	1	0.902	0.902
b (m)	0.5	0.5	0.5
Cost (U.P)	754.649	884.886	929.505
FE*	141	44	44
SF	4.104	5.398	5.396
SET (m)	$2.3963*10^{-06}$	$1.986*10^{-06}$	$1.986*10^{-06}$
SCD (m)	$8.229*10^{-08}$	$3.257*10^{-08}$	$3.257*10^{-08}$
MM** (kN.m)	-1165.531	-1374.445	-1374.445

* : FE = number of function evaluation

** : MM = maximum negative bending moment along footing base

Table (2) The Design Results (initial trial points)

Variables	First trial point	Second trial point	Third trial point
B (m)	2.13	2.14	2.12
T (m)	0.42	0.42	0.42
DF (m)	1.0	0.75	1.0
b (m)	1.0	0.75	0.5
Cost (U.P)	765.117	766.817	754.649
FE*	145	141	141
SF	4.182	4.063	4.104
SET (m)	$2.384*10^{-06}$	$2.3708*10^{-06}$	$2.396*10^{-06}$
SCD (m)	$8.017*10^{-08}$	$7.806*10^{-08}$	$8.229*10^{-08}$
MM** (kN.m)	-1171.029	-1176.526	-1165.531

Table (3) The Results of Analysis Associated with the Angle of Internal Friction

Variables	Angle of Internal Friction, ϕ (degrees)				
	24	25	26	30	40
B (m)	2.354	2.16	2.12	2.12	2.12
T (m)	0.439	0.42	0.42	0.42	0.42
DF (m)	1.0	1.0	1.0	1.0	1.0
b (m)	0.5	0.5	0.5	0.5	0.5
Cost (U.P)	930.248	779.078	754.649	754.649	754.649
FE*	146	141	141	141	141
SF	2.003	2.014	4.014	4.014	4.014
SET (m)	$2.126*10^{-06}$	$2.345*10^{-06}$	$2.396*10^{-06}$	$2.396*10^{-06}$	$2.396*10^{-06}$
SCD (m)	$4.508*10^{-08}$	$7.403*10^{-08}$	$8.229*10^{-08}$	$8.229*10^{-08}$	$8.229*10^{-08}$
MM** (kN.m)	-1294.179	-1187.522	-1165.531	-1165.531	-1165.531

Table (4) The Results of Analysis Associated with the Load

Variables	Load, P (kN)				
	775	825	875	925	975
B (m)	1.989	2.05	2.12	2.18	2.23
T (m)	0.419	0.439	0.42	0.43	0.46
DF (m)	1	1.0	1.0	1.0	1.0
b (m)	0.5	0.5	0.5	0.5	0.5
Cost (U.P)	635.458	695.521	754.649	809.00	871.67
FE*	143	146	141	141	146
SF	2.276	4.056	4.104	4.162	4.141
SET (m)	$2.284*10^{-06}$	$2.349*10^{-06}$	$2.396*10^{-06}$	$2.440*10^{-06}$	$2.519*10^{-06}$
SCD (m)	$1.031*10^{-08}$	$9.347*10^{-08}$	$8.229*10^{-08}$	$7.232*10^{-08}$	$6.868*10^{-08}$
MM** (kN.m)	-968.537	-1062.644	-1165.531	-1272.816	-13.66.121

Table (5) The Results of Analysis Associated with the Diameter Ratio

Variables	Diameter Ratio, D_{cl}/B				
	0.04	0.08	0.12	0.16	0.2
B (m)	2.12	2.12	2.12	2.12	2.12
T (m)	0.45	0.44	0.42	0.419	0.42
DF (m)	0.9	0.9	1.0	1.0	1.0
b (m)	0.5	0.5	0.5	0.5	0.5
Cost (U.P)	769.535	769.505	754.649	747.69	749.43
FE*	146	146	141	141	141

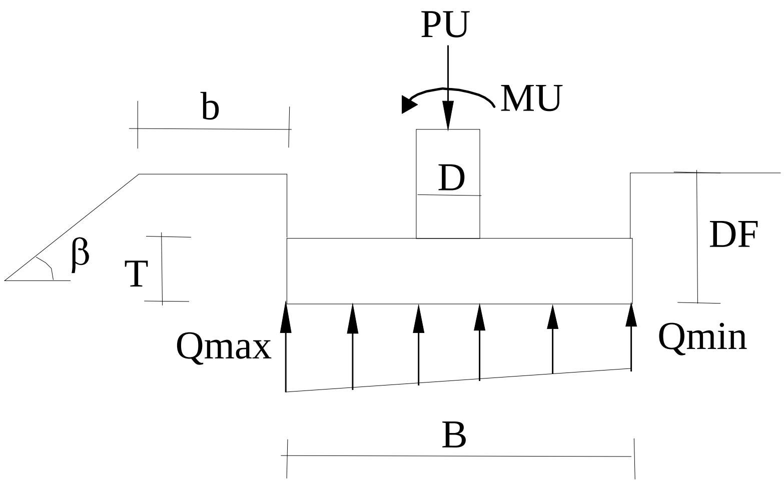

Fig. (1) Forcec on circular footing adjacent to slope

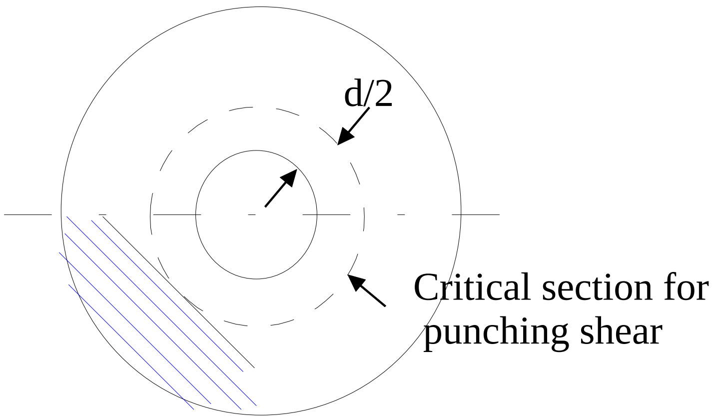

Fig. (2) Critical section for punching shear
for circular footing [5]

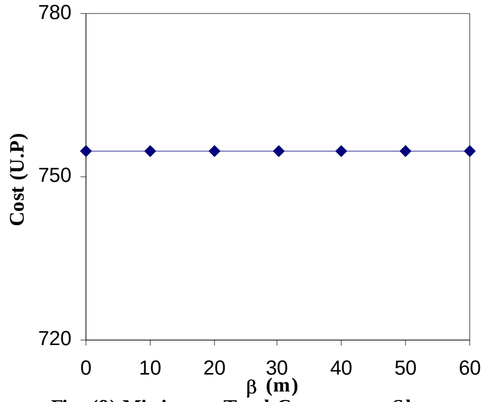

Fig. (9) Minimum Total Cost versus Slope Angle

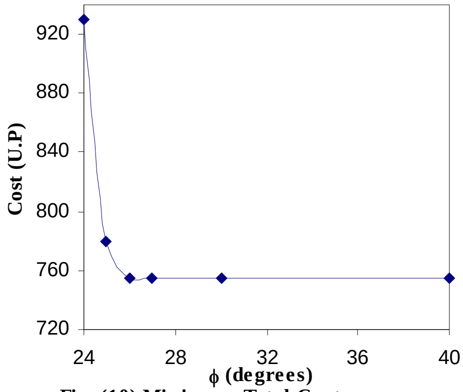

Fig. (10) Minimum Total Cost versus Friction Angle

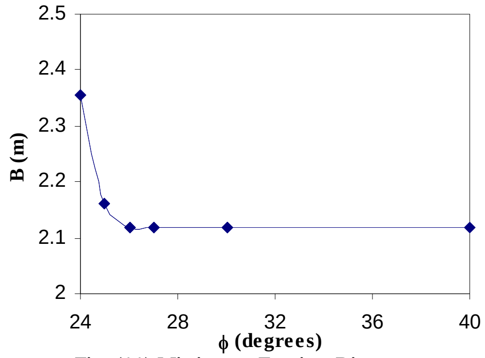

Fig. (11) Minimum Footing Diameter versus Friction Angle

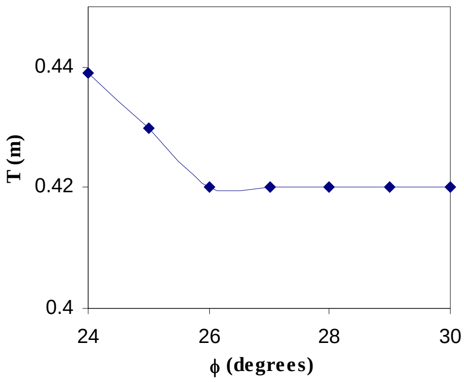

Fig. (12) Minimum Footing Thickness versus Friction Angle

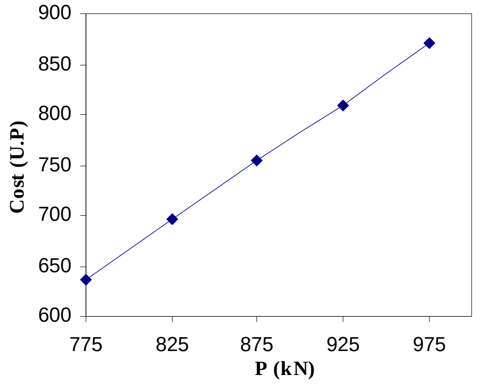

Fig. (13) Minimum Total Cost versus Column Load

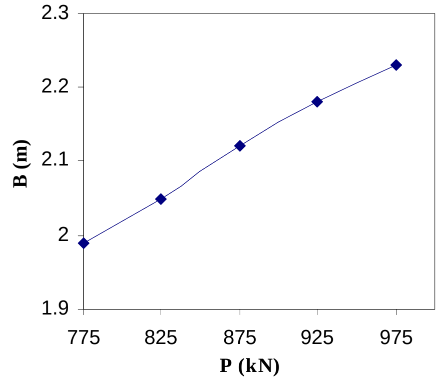

Fig. (14) Minimum Footing Width versus Column Load

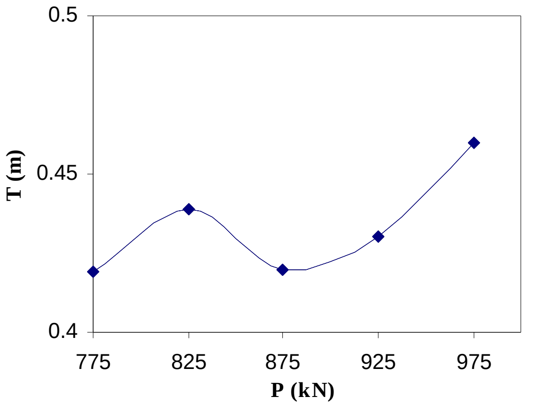

Fig. (15) Minimum Footing Thickness versus Column Load

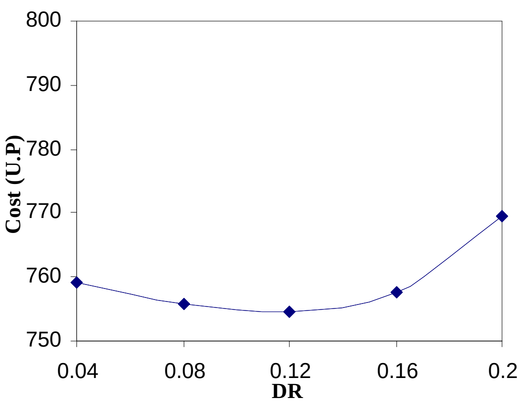

Fig. (16) Minimum Total Cost versus Diameter Ratio

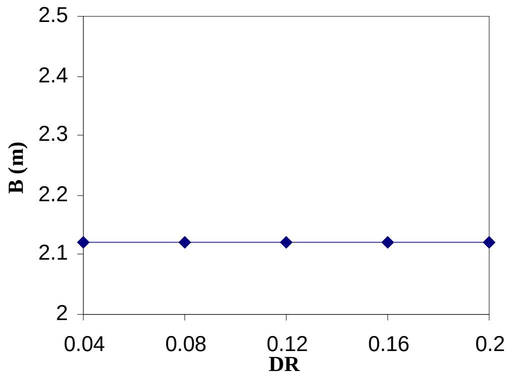

Fig. (17) Minimum Footing Diameter versus Diameter Ratio

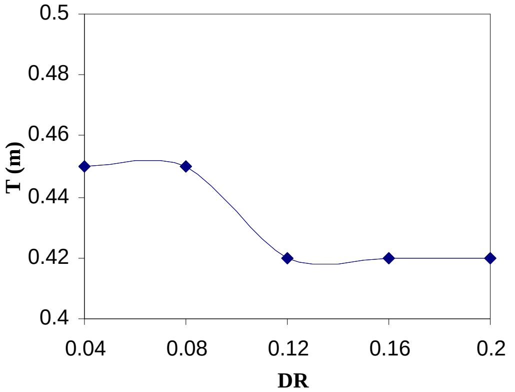

Fig. (18) Minimum Footing Thickness versus Diametr Ratio

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قسم الهندسة المدنية جامعة تكريت

الخلاصة

تتعلق هذه الدراسة بثلاث من طرق البرمجة اللاخطيــة (Non-linear Programming Method) وبالتحديــد طريقة هوك وجيفز Hooke & Jeeves Method وطريقة فليتشر -ريفز Fletcher-Reeves Method وطريقة دافدن -فليتشر -باول Davidon-Fletcher-Powell Method على مسألة التصميم الإنشائي للأســس الدائريــة المجاورة للمنحدرات على تربة رملية، على اعتبار ان الكلفة الكلية للأساس هي دالة الهدف. صيغت دالة الهــدف بدلالة المتغيرات التصميمية التالية (عرض الأساس، سمك الأساس، عمق الدفن، بعد حافة الأساس عن المنحدر).

أعد برنامج حاسبة لحل هذه المــسألة التــصميمية باســتخدام طريقــة التــصميم الإنــشائي التقليــدي (The Conventional Structural Design Approach) بالارتباط مع طرق بحوث العمليات المشار إليهــا أعلاه.

نفذت دراسة بسيطة لتحري مدى حساسية دالة الهدف إزاء متغيراتها التصميمية كمــا أجريــت دراســة أعمق لبيان تأثير زاوية الاحتكاك الداخلي للتربة و ظروف التحميل و نسبة عرض العمود إلى عــرض الأســاس على الكلفة الكلية. ووجد من خلال هذه الدراسة أن الكلفة الدنيا للأساس تتزايد مع زيادة زاوية الاحتكاك إلى حــد زاوية مقدارها 26 درجة حيث تثبت الكلفة بعدها كما تتزايد تبعا لزيادة نسبة التحميل.

الكلمات الدالة : الأمثلية، تصميم تقليدي ميل، أساس دائري، تربة رملية.