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# *Response Analysis of Electro-Optic Electric Field Sensor*

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**Abstract**

In this paper an electric field sensor based on the electro-optical effect in Lithium Niobate crystal is studied. The electro-optically induced polarization modification in crystal has been described and the response analyzed for different crystal lengths and light source wave lengths. The study shows that as the crystal length increased the required electric field to produce a phase-shift equal $\pi$ is decreased. The responsivity of the sensor for different ranges of the electric field to be measured has been calculated and it is found that the rate of change of the half of the phase shift with respect to the electric field $d(\phi/2)/dE$ is equal to the responsivity of the sensor at the mid-point of the linear part of the light intensity response curve.

**Key words: Electro-Optic, Electric field, Field sensor**.

## تحليل استجابة متحسس المجال الكهربائي الكهروبصري

**الخلاصة**

في هذا البحث تم دراسة متحسس المجال الكهربائي المبني على أساس التأثير الكهروبصري في بلورة الليثوم نايوبيت. إن تعديل الاستقطاب المستحث بشكل كهروبصري تم وصفه وتحليل الاستجابة كدالة لأطوال مختلفة للبلورة والطول الموجي لمصدر الضوء. لقد أظهرت الدراسة ان المجال الكهربائي اللازم لتوليد فرق طور مقداره 180 درجة يتناقص بزيادة طول البلورة. إن استجابية المتحسس لمديات مختلفة من المجال الكهربائي المراد قياسه تم حسابها ولقد وجد بأن معدل التغير لنصف فرق الطور نسبة للمجال الكهربائي $d(\phi/2)/dE$ يساوي استجابية المتحسس في منتصف الجزء الخطي لمنحني استجابة شدة الضوء.

**الكلمات الدالة:** الكهروبصري، المجال الكهربائي، متحسس المجال.

## Introduction

An electric field sensor based on the electro-optical effect (pockels effect) in Lithium Niobate ($LiNbo_3$) crystal has been studied to meet the requirements for many electric field applications.

The optical sensor could be used with fiber optics for suitable length to enable the measuring devices to be immune from any interference produced by the high voltage system. The sensor head must be small in order to keep the distortion of the electric field to be measured as minimum as possible. The bandwidth should be more than 1 GHz for transient applications.

An ideal electro-optical crystal for electric field measurements should have the following physical characteristics [1].

1- Ruggedness.
2- Small electric permeability.
3- Large electric resistance.
4- No Pyroelectricity.
5- No optical activity.
6- No natural birefringence.

Lithium Niobate crystal posses many of the above physical characteristics so it is suitable choice to be used as a sensor for electric field. Lithium Niobate ($LiNob_3$) is compound of niobium and Lithium. It is a colorless solid material with trigonal crystal structure. It is transparent for wavelengths between 350-5500 nanometers and band-gap of 4 ev. It's melting point is 1257 °C. It is insoluble in water [2].

The crystalline form is electro-optic and birefringent which mean that it modifies the polarization of transmitted light in response to an applied electric field.

Hence by inserting a crystal of lithium Niobate between crossed polarizer (their transmission axes at right angles) or aligned polarizer (their transmission axes in the same direction), the transmitted intensity response function with applied field is a $\sin^2(\phi/2)$ or $\cos^2(\phi/2)$ respectively. The orientation of the crystal was chosen such that the light propagates along the length of the crystal in the z-direction (optical axis) and the electric field applied along the x-direction.

The input light is provided by a source of light (laser) with different wavelengths from the infrared region to the visible region. The field alteration is carried by the intensity of the transmitted light and delivered to a high bandwidth photo detector.

The values of the refraction index n and the electro-optical coefficient r related to the wavelengths of the light source λ are listed in the table(1) [2].

**Theory**

The electro-optical effect is a change in the index of refraction for certain crystals as a function of applied electric field. The index change is dependent on the direction and polarization of the incident light beam [3].

The function between the index of refraction and the applied field varies slightly and can be expanded in a Taylor's series and approximated to the following equation [4].

$$n - n(E) = \Delta n \approx \frac{1}{2} r n^3 E \qquad \text{.....(1)}$$

n : Refraction index of the crystal.
r : Electro-optic coefficient of the crystal.
E : The applied field along the x-direction.

The electro-optic coefficient depends on the direction of the applied electric field and the polarization of the incident light. Typical value of the electro-optical coefficient lies in the range of $10^{-12}$ to $10^{-10}$ m/V. The electro-topic effect leads to the ability of controlling the light intensity that pass through the crystal by changing the applied field across the crystal.

According to equation (1), when an electric field applied on the crystal along the x-direction, the index of refraction decreased a long that direction, while the index of refraction along the y-direction increased. The velocity of the light beam inside the crystal is:

$$v = \frac{c}{n} \qquad \text{........ (2)}$$

Where c : The velocity of light in the free space.

Therefore as n decreases along the x-direction, the velocity of the light component polarized along that direction increase. A beam of light polarized along x-direction propagates with speed faster than a beam of light polarized along the y-direction, consequently the x-axis is known as the fast axis and the y-axis known the slow axis[5].

The two components of the light travel in the same direction through the crystal and do not become physically separated, but the two components in phase as they enter the crystal, emerge with different phases. As they traverse the crystal they, accumulate a phase difference which depends on the distance traveled inside the crystal and on the applied field [3].

The phase shift of the light wave is given in equation(3) [6].

$$\Delta\phi = \frac{2\pi}{\lambda} L\,\Delta n \qquad \ldots (3)$$

Where L: The crystal length.

$\lambda$: The wavelength of the light source.

The accumulated phase difference between the two components of the light beam with respect to each other is:

$$\Delta\phi = \Delta\phi_y - \Delta\phi_x$$

$$\Delta\phi = \frac{2\pi L}{\lambda}\left(\Delta n_y - \Delta n_x\right) \qquad \ldots (4)$$

Where:

$\Delta\phi_y$: the phase shift of the y-components.

$\Delta\phi_x$: the phase shift of the x-components.

$\Delta n_y$: the value of the refraction index change along the y-axis.

$\Delta n_x$: the value of the refraction index change along the x-axis.

The value of $\Delta\phi_x \approx -\Delta\phi_y$ since they have a similar expression except that r changes sign [5].

From equation (1) The total phase-shift between the two components of the light is.

$$\therefore \Delta\phi = \frac{2\pi n^3 r L\, E}{\lambda} \qquad \ldots\ldots\ldots\ldots\ (5)$$

The electric field required to maintain a phase-shift equal $\pi$ between the two components of the light is called the half-wave electric field $E_\pi$. The factor $E_\pi$ is a significant feature of the crystal it depends on the refraction index, electro-optical coefficient, the wavelength $\lambda$ and the crystal length L.

The phase-shift can be expressed by the half-wave electric field as follow:

$$\Delta\phi = \pi \frac{E}{E_\pi} \qquad \ldots\ldots\ldots\ldots\ldots (6)$$

Where: $E_\pi = \dfrac{\lambda}{2 n^3 r L}$ in V/m.

Phase delay alone does not affect the intensity of a light beam However; a crystal placed between two polarizer can be considered as an intensity controller.

By sandwiching a crystal between two crossed polarizer and the orientation of the polarizer and analyzer at 45° to the vertical line as shown in Fig.(1). The polarization vector is composed of two perpendicular components of equal intensity, vertical and horizontal. The two components travel with different velocities inside the crystal. The response function of the optical sensor for this configuration is given by Malus's law[7]:

$$I_T = I_o \,\mathrm{Sin}^2\left(\frac{\Delta\phi}{2}\right) \qquad \ldots\ldots\ldots\ldots (7)$$

Where $I_T$ is the transmitted intensity of the light and $I_o$ is the incident light intensity after the first polarizer.

From equations (6) and (7) the light intensity is

$$I_T = I_o \,\mathrm{Sin}^2\left(\frac{\pi}{2}\frac{E}{E_\pi}\right) \qquad \ldots\ldots (8)$$

The response as a function of the applied field is shown in Fig.(2). For zero electrical field, no light can be detected after the analyzer because of no change occurs on the linearly polarized light that emerge out of the first polarizer. When the field equal $E_\pi$, the intensity of the light is maximum $I_T = I_o$. If the system required to operate at the linear part of the response curve a quarter-wave plate (QWP) should be inserted between the polarizer and the crystal to generate a phase-shift equal π/4 between the two components of light that is entered the crystal and they became circularly polarized instead of the plane polarized waves and the system will be biased at point Q as in Fig.(2). The applied field on the crystal will induces elliptically polarized light and the response function will be:

$$I_T = I_o \, Sin^2 \left( \frac{\pi}{2} \frac{E}{E_\pi} + \frac{\pi}{4} \right) \qquad ...... (9)$$

The components of the light that turn out to be in the same direction of the transmission axis of the analyzer will exceed into the photo detector which gives light intensity equal 0.5 $I_o$ for zero applied electric field and $I_T = I_o$ for electrical field equal $E_\pi$.

**Responsivity and Ranges of the Sensor**

The responsivity of the sensor is the gradient of the linear part of the response curve. It depends on the values of the half-wave electric field $E_\pi$.

The range of the electric field to be measured is the difference between the upper limit and the minimum detectable field (Lower limit), and depends on the geometry of the crystal and the light source wavelengths.

For crossed polarizer and a quarter-wave plate positioned between the polarizer and the crystal the maximum electric field can be measured is equal to 0.5 $E_\pi$ which present half the light transmittance intensity for zero electric field and full transmittance for electric field equal $E_\pi$ as shown in Fig.(2) [8].

To obtain a large range used for high electric field measurements, $E_\pi$ should be large and this can be done by choosing a crystal of short length or long wavelength for the light source (red or infrared region).

The ranges have a small percent of $E_\pi$ (0.05 – 0.35), the deviation of the real response curve from linearity is very small and can be approximated to zero but for ranges more than 0.35 $E_\pi$ to 0.5 $E_\pi$ the deviation is large and this leads to lose the accuracy.

The lower limit of the electric field to be measured depends on the minimum light intensity that can be detected by the photo detector $I_m$ which is a very small percent of $I_o$, therefore for a sensor responsivity R, the lower limit of the electric field is $\mathbf{E_l = I_m/(I_o\ R)}$.

**Analysis and Discussion**

***The half-wave electric field $E_\pi$***

The relation between the half-wave electric field $E_\pi$ and the crystal length is shown in Fig. (3) and Fig.(4). It is shown that as the crystal length increased, the value of $E_\pi$ reduces. This because the accumulated nature of the phase-shift between the components of the light that pass through the crystal. From Fig.(5) and for λ =632.8 nm, the value of $E_\pi$ = 65 KV/m for crystal length L = 60 mm which give a phase shift equal π , thus the phase shift for each 10mm long from the crystal length is π/6 for the same value of $E_\pi$. From Fig.(5) also for crystal length L=10 mm, the value of the applied electric field required to produce a phase shift equal to π/6 (point A3) is 65KV/m which is the same value of $E_\pi$ for crystal length L = 60 mm. It means that the necessary electric field required to give a phase-shift equal to π in a crystal with length L = 10mm is six time greater than that one needed for crystal length L=60mm.

The values of $E_\pi$ enlarges with the increase of the wavelength of the light source as shown in Fig.(3) and Fig.(4) because the values of the refraction index diminishes with the increasing of the wavelengths of the light source as shown in the Table(1). As mentioned before the phase-shifted is accumulated as a result of the interaction between the applied electric field and the light wave that pass through the crystal. This interaction depends on the wave number of the light and it will be more significant for long wavelengths.

**Responsivity**

The responsivity of the optical sensor deduced from the gradient of the response curve is shown in Fig.(6), Fig.(7) and Fig.(8). For each crystal length there is one value of $E_\pi$ , therefore the x-axis in these figures characterized as a percentage of the $E_\pi$ for each crystal length. The responsivity increased as the crystal length increased and the reason mentioned earlier as a result of decreasing $E_\pi$. When $E_\pi$ get large value the slope (gradient) of the response curve (light intensity with the electric field curve ) is low, while becomes high for low values of $E_\pi$.

The responsivity is also increased as the wavelength of the light source decrease because of the linearity between $E_\pi$ and $\lambda$. The responsivity calculated for different ranges of electric field and it is found that the best values can be obtained for ranges lie in the linear section of the response curve for values from (0.05 – 0.35) of the values of $E_\pi$ as shown in Fig. (6), Fig. (7) and Fig. (8).

Anew approach is used to calculate the responsivity of the optical sensor at the mid-point of the light intensity response curve, by determining the rate of change of the half of the phase-shift with respect to the applied field $d(\phi/2)/dE$ and the result are shown in Fig. (9). The results are the same as those obtained for ranges of very small percent of $E_\pi$ as in Figs. (6-8).

**Band width**

The electro-optical effect is very fast, therefore the bandwidth is very high even for the long crystal L = 60 mm. The transient time needed to pass through the crystal is affecting the bandwidth which is more than 2GHz for the longest transient time. The wavelength of the light source has no great effect on the bandwidth as shown in Fig.(10) for $\lambda = 632.8$ nm and $\lambda = 3391.3$ nm because they almost have the same values for the corresponding refraction indices.

**Ranges of the electric field**

To specify the ranges of electric field to be measured the upper limit should be known and its value should not exceed 0.35 of the half-electric field value $E_\pi$ in order to keep the measurements with acceptable errors. From $E_\pi$ the responsivity can be calculated and finally the lower limit can be determined. The lower limit mostly equal zero unless the minimum light intensity can be detected by the photo detector $I_m$ represented noticeable percentage of the maximum light intensity $I_o$. Table(2) shows the whole values for ranges 30 KV/m and 70 KV/m for $\lambda$=632.8 nm and the minimum light intensity assumed 0.001 the value of the maximum intensity ( $I_m$=0.001 $I_o$). The value of the ratio of the upper limit electric field $E_u$ to the half-wave electric field $E_\pi(E_u/E_\pi) \leq 0.35$, this gives which responsivity is suitable and then the lower limit electric field can be calculated.

**Conclusions**

The sensing element of an optical electric field sensor using the electro-optical effect of Lithium Niobate crystal has been studied. The study presented the response analysis of six types of crystal depending on the effect of the crystal lengths and the wavelengths of the light source on the electric field required to produce a phase-shift equal to π. Emphasis has been given on calculations of the responsivity of the optical sensor as a function of the half-wave electric field $E_\pi$. The best ranges for measuring electric field with high responsivity and bandwidth in the linear part of the response curve near the mid-point. If the range is increased, the upper limit of the measured electric field deviated from linearity and this leads to errors in the measurements.

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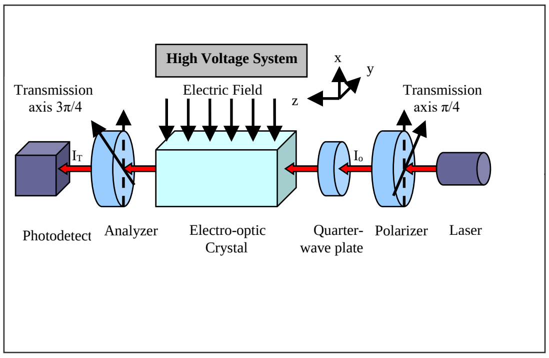



**Figure (1) Schematic diagram of the electro-optical sensor components**

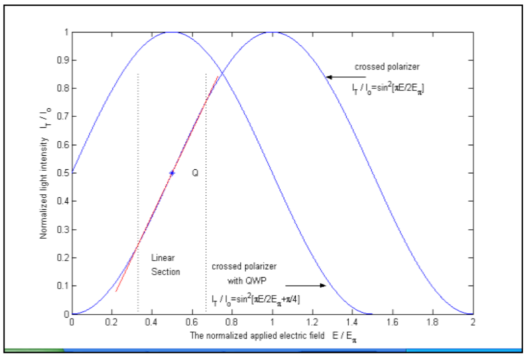


**Figure (2) The response curve of the optical sensor for crossed polarizes**

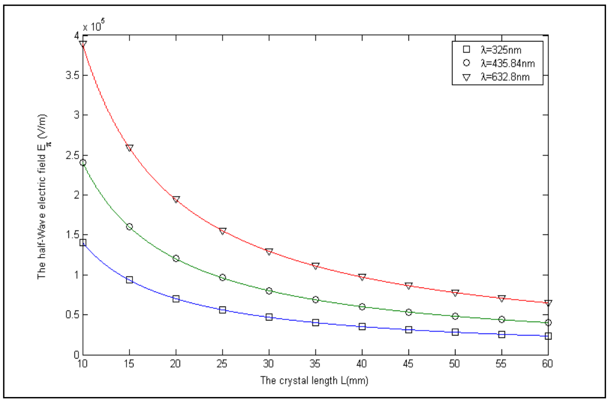
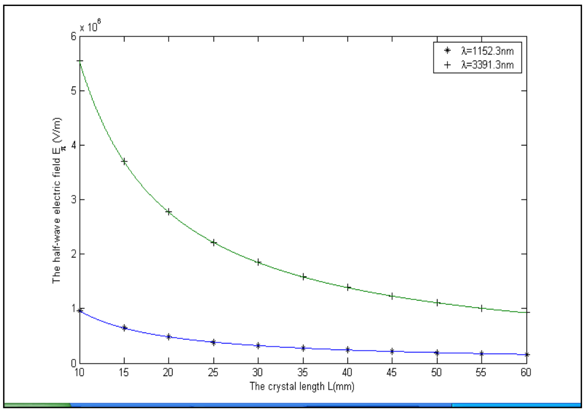
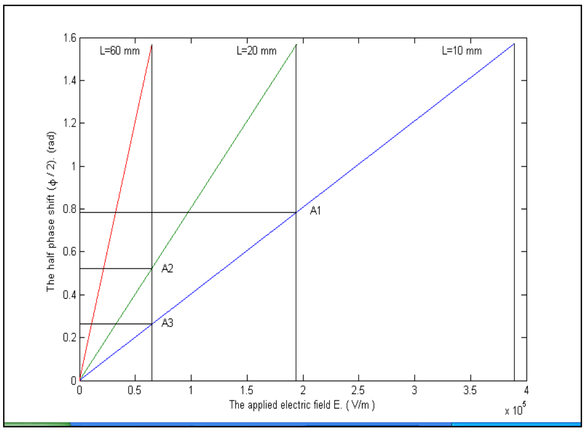


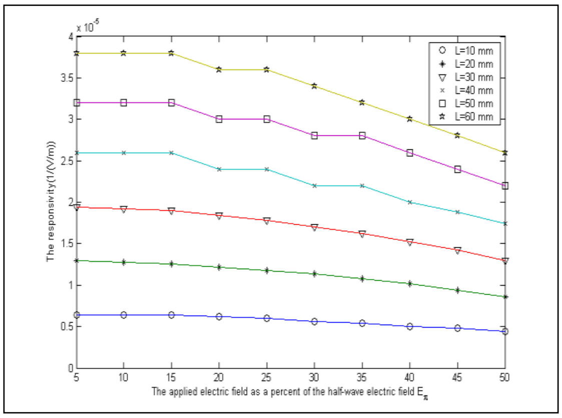

**Figure (3) The half-wave electric field $E_\pi$ versus crystal length L for $\lambda \leq 632.8$ nm**



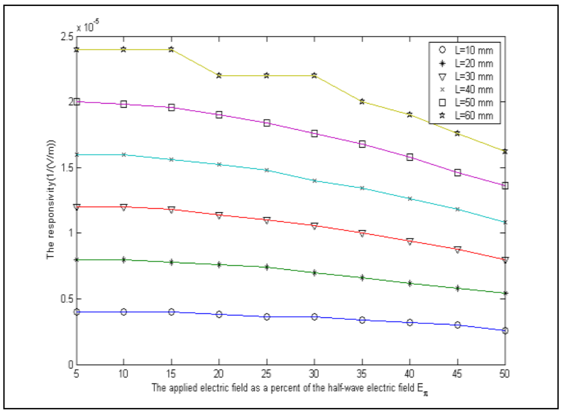

**Figure(4)The half-wave electric field $E_\pi$ versus crystal length L for $\lambda$ in the infrared region**



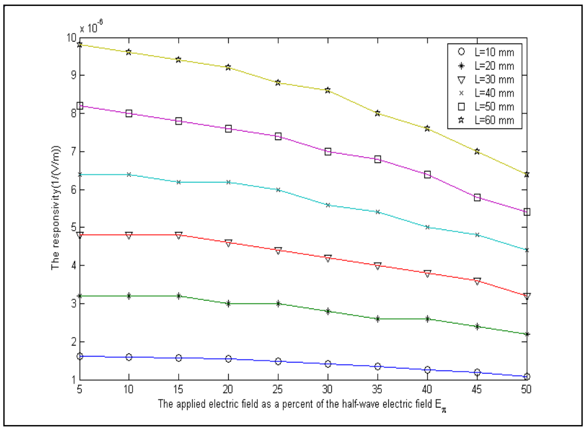

**Figure(5)The half phase shift ($\phi$ / 2) versus the applied electric field for $\lambda = 632.8$nm**



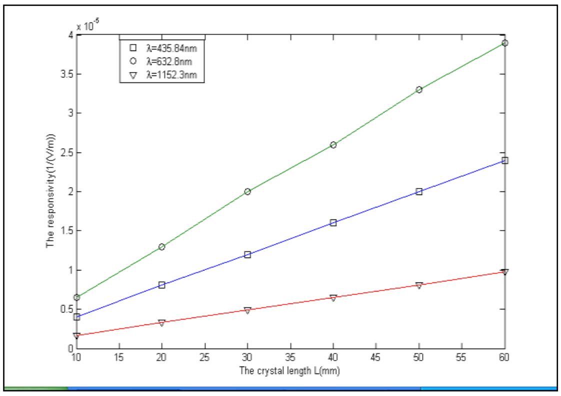

**Figure(6)The responsivity versus the applied field as a percent of $E_\pi$ for $\lambda$=435.84nm**



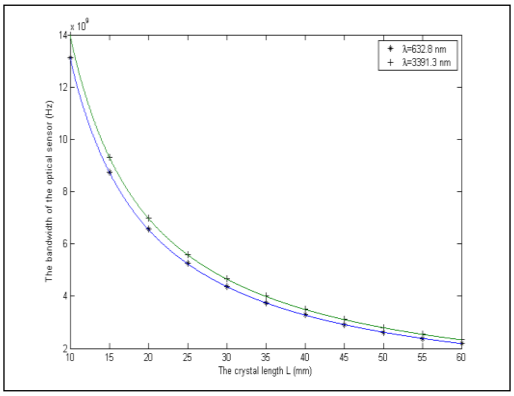

**Figure(7)The Responsivity versus the applied field as a percent of $E_\pi$ for $\lambda$=632.8nm**




**Figure(8)The Responsivity versus the applied field as a percent of $E_\pi$ for $\lambda$=1152.3nm**




**Figure(9)The Responsivity versus the crystal length**




**Figure(10)The bandwidth of the optical sensor versus crystal length**

**Table(1) Electro-Optic Electric Field**

| (nm)λ | n | r (pm/V) |
|---|---|---|
| 325 | 2.2636 | 6.3 |
| 435.84 | 2.393 | 6.6 |
| 632.8 | 2.286 | 6.8 |
| 1152.3 | 2.227 | 5.4 |
| 3391.3 | 2.145 | 3.1 |

**Table (2) whole values for ranges 30 KV/m and 70 KV/m for λ=632.8 nm**

| λ = 632.8 nm | | $E_u$ = 30 | | | $E_u$ =70 | | |
|---|---|---|---|---|---|---|---|
| L | $E_\pi$ | $E_u / E_\pi$ | R | $E_l=I_m/I_oR$ | $E_u / E_\pi$ | R | $E_l=I_m/I_oR$ |
| 10 | 390 | 0.08 | 4 | 0.25 | 0.18 | 3.8 | 0.26 |
| 20 | 195 | 0.15 | 7.8 | 0.13 | 0.35 | 6.6 | 0.15 |
| 30 | 130 | 0.23 | 11 | 0.09 | 0.54 | ---- | ---- |
| 40 | 97.5 | 0.3 | 14 | 0.07 | 0.72 | ---- | ---- |
| 50 | 78 | 0.38 | ---- | ---- | 0.9 | ---- | ---- |
| 60 | 65 | 0.46 | ---- | ---- | 1.08 | ---- | ---- |

$E_u$ – Uper limit of the electric field to be measured (KV/m).
$E_l$ – Lower limit of the electric field to be measured (KV/m).
$I_o$ – The maximum light intensity.
$I_m$ –The minimum light intensity (0.001 $I_o$)
$E_\pi$ –Half-wave electric field (KV/m).
R – Responsivity (1/(MV/m)).
L – Crystal length (mm).