ANALYSIS OF THE HARMONIC ORDER AFFECTING THE EDDY CURRENT BRAKING FORCE IN ELECTRICAL MACHINES

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ABSTRACT

This paper presents a computer analysis of the eddy current brake in electric machines. It presents a formula for the braking force when the actual width of the pole is considered. This formula is suitable for both thin and thick discs and may be employed for a wide range of working speed. For this purpose, a mathematical analysis of the problem is presented together with the formula achieved for the braking force. The brake is first represented by a mathematical model based on certain assumptions and then the braking force is obtained as a result of solving a field problem. The problem is simplified to a one-dimensional problem, where a solution for the magnetic vector potential is obtained, and by employing Lorentz force equation, a formula for the braking force of the nth harmonic order is obtained.

KEYWORDS: Electric Machines, Eddy Currents, Braking Force, Electromagnetic Fields, Poisson's equation.

NOTATIONS

Ag, Ap	Magnetic vector potential of the air gap region and the plate.
A_n	vector potential of the nth harmonic
Fb_n	Braking force of the nth harmonic
2g	Air gap length
J(x)	Current density
J_0	Amplitude of current density
J_n	Current density of the nth harmonic
n	Harmonic order
Pn, Qn, Rn, Sn,	Constant obtained by using appropriate boundary conditions
t	Time
V	Velocity of the plate
x, y, z	Stationary coordinate system
α,β	Attenuation and phase shift constant
2δ	Plate thickness
μ	Permeability
$\mu 0$	Permeability of free space

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σ Electrical conductivity

τ Pole pitch

* Denotes complex conjugate of vector quality

INTRODUCTION

The idea [1] of eddy current stems from the fact that when a metal moves through a spatially varying magnetic field, or is located in a changing magnetic field, induced currents begin to circulate through the metal. These currents are called eddy currents because of their similarity to eddies in a flowing stream. In the case of the eddy current brake, a rotating disk has a magnetic field passing through it perpendicularly, but it is only strong in the area where the magnetic is .The currents in that area experience a side thrust, which opposes the rotation of the disk. This interaction of field and current results in the "braking" of the disk, and thus the name "eddy current brake". The return currents close via parts of the disk where the field is weak, so there is a drag force only in the "generating" region.

One early study into eddy current braking was performed by Davis and Reitz [2](1971) , who examined the forces induced on a magnet moving over the surface of both a semi-infinite and a finite conducting medium. Later, Schieber [3](1974) analytically predicted the braking torque on a finite rotating conductive sheet and performed experiments to identify the accuracy of the modeling techniques. Schieber continued his research in eddy current braking and published a paper [4](1975) that analytically found the optimal size of a rectangular electromagnet for eddy current braking. Venkatanatnam and Ramachandra [5](1977) obtained the field distribution when a thin conducting sheet moves with constant velocity between two rectangular and infinitely permeable pole pieces of an electromagnet. Nagaya et al. [6](1984) investigated the eddy current damping force induced on a conducting plate of arbitrary finite size moving with a velocity parallel to the face of a cylindrical magnet. Wiederick et al. [7](1987) proposed a simple theory for the magnetic braking force induced by eddy current in a thin rotating conductive disk passing through the poles of an electromagnet. Cadwell [8](1996) investigated the braking force exerted on an aluminum plate as it passes between the poles of a horseshoe electromagnet

Lee and Park [9](1999) investigated the design of an eddy current brake controller. However, their system was not intended to maintain a fixed speed, but to minimize the stopping time. More recently, Lee and Park [10](2001a) and Lee, K. [11](2002) have developed a model for an eddy current braking system that allows for an analytical solution to the problem. In this paper, to investigate the harmonic order effecting on the braking force in electrical machine, a formula for the braking force has to be developed first. Calculations of the braking force in eddy current brakes has received by employing Lorentz force equation, a formula for the braking force of the nth harmonic order is obtained.

Mathematical Model Assumptions

A complete 3-dimensional solution for the braking force is difficult to obtain. Instead, the configuration of a quasi one-dimensional model is used as shown in Fig.(1). This model is obtained under the following assumptions: -

1. The plate and pole structure are considered to be infinitely wide in the y-direction, so that all variables become independent of y; hence $\frac{\partial}{\partial y} = 0$

In the following analysis, the transverse edge effects are taken into account.

2. All currents are y-directed.

3. The excitation winding and the salient poles are replaced by infinitely thin current sheets backed by smooth iron boundaries. These linear current sheets are chosen in such away that they cover only a pole face only and give the same field in the air gap of the model having smooth structures as the original windings produced in the actual machine.

4. The pole material has infinitely larger permeability $(\mu_{ri} \to \infty)$ and small electrical conductivity $(\sigma i \to 0)$.

5. The plate and pole structures are assumed to be very long in the $\pm x$ direction. Hence, the longitudinal end effects are neglected.

Formulation of Equation And Solution of The Problem

In the actual problem, the plate is moving while the pole structure is stationary. The current distribution for an observer on the structure takes a repeatable step distribution as shown in Figure (2). This distribution when expressed by Fourier series gives [5]:

$$j(x)=\sum_{n=1}^{\infty} j_n \qquad \sin nx\frac{\pi}{\tau}=\sum_{n=1}^{\infty} j_n \qquad \mathrm{Re}\left[-je^{j}\frac{\pi}{\tau}nx\right]$$

----------- (1)

Where j_n is current density of nth harmonic, which is represented by:-

$$j_n=\frac{4 \quad jo}{n\tau}\left[\sin\frac{n\tau}{2}\sin\frac{n\tau'}{2}\right]$$

n:- harmonic order.

If the plate is considered stationary and the field system moves in the (-ve) direction of x, then the current density function takes the form:-

$$j(x,t)=\sum_{n=1}^{\infty} j_n \qquad \mathrm{Re}[-je^{jp(nx+vt)}]$$

----------- (2)

Where $p=\frac{\pi}{\tau}$, v: the velocity of the plate.

The differential equation for the vector potential in the plate is:

$$\frac{\partial^2 Ap}{\partial X^2}+\frac{\partial^2 Ap}{\partial Z^2}=\mu\sigma\frac{\partial Ap}{\partial t}$$ (Poisson's formula) ----------- (3)

For the air gap σ=0 , and Equation 3 becomes:

$$\frac{\partial^2 Ap}{\partial X^2}+\frac{\partial^2 Ap}{\partial Z^2}=0$$ ---------- (4)

The excitation and boundary conditions require a vector potential which depends on x & t as $e^{j(nx+vt)}$

Assuming this solution to be on the form:

$$\mathrm{A}(\mathrm{x},\mathrm{z},\mathrm{t})=\sum_{n=1}^{\infty} A_n$$ ---------- (5)

Where $\mathrm{A}_n=\mathrm{A}_n(\mathrm{z})\mathrm{e}^{\mathrm{jp(nx+vt)}}$ ---- (6)

Substituting 6 in 3 & 4 gives for the nth component of the vector potential.

$$\frac{d^2Ap_n(Z)}{dZ^2}-g_n^2 \qquad Ap_n(Z)=0$$ - (7)

Where:

$$g_n=p_n(1+j\frac{\sigma v\mu}{p_{n^2}})^{1/2}$$

And $$\frac{d^2Ap_n(Z)}{dZ^2}-p_n^2 \qquad Ag_n(Z)=0$$

---------- (8)

Where $p_n=n*p$

The general solution of Equations 7 & 8 are:

$$Ap_n(z)=p_ne^{g_nz}+Q_ne^{-g_nz}$$ ---- (9)

$$Ag_n(z)=R_ne^{p_nz}+S_ne^{-p_nz}$$ ------ (10)

Where p_n, Q_n, R_n and S_n are arbitrary constants which can be found by applying the boundary conditions.

The Boundary Conditions

A set of conditions, which are important in determining the

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mathematical solution to many physical problems, specified for the behavior of the solution to a set of differential equations at the boundary of its domain.

1. At z =0, the differential of vector potential of the plate for n harmonic with respect to. z = 0

i.e $\frac{\partial Ap_n}{\partial z} = 0$.

2. At z= δ (plate thickness)

The vector potential of the air gap equal the vector potential of the plate.

i.e A p_n= A g_n

3. $\frac{1}{\mu}\frac{\partial Ap_n}{\partial Z} = \frac{1}{\mu_0}\frac{\partial Ag_n}{\partial Z}$.

4. At z = g (air gap length)

The differential of vector potential of the gap for n harmonic with respect to z = k_n

i.e $\frac{1}{\mu_0}\frac{\partial Ag_n}{\partial Z} = k_n$.

From Condition 1

$$P_n g_n - Q_n g_n = 0$$
$$i.e. \quad P_n = Q_n$$

Hence A $P_n(z)=2p_n \cosh g_n z$ --- (11)

From Condition 2

$$P_n e^{\delta g_n} + Q_n e^{-\delta g_n} = R_n e^{\delta p_n} + S_n e^{-\delta p_n}$$

$$2P_n \cosh \delta g_n = R_n e^{p_n \delta} + S_n e^{-p_n \delta} \quad (12)$$

From Condition 3

$$g_n (p_n e^{\delta g_n} - Q_n e^{-\delta g_n}) = P_n (R_n e^{\delta p_n} - S_n e^{-\delta p_n})$$

$$2P_n \gamma_n \quad \sinh \quad g_n \delta = R_n e^{\delta p_n} - S_n e^{-\delta p_n} \text{ - (13)}$$

Where $\gamma_n = g_n / p_n$

From Condition 4

$$R_n e^{p_n g} - S_n e^{-p_n g} = \frac{k_n \mu_0}{p_n} \quad \text{---------- (14)}$$

Adding 12 &13 gives

$$R_n = p_n \{\cosh \quad \delta g_n - \gamma_n \sinh \delta g_n\} e^{-p_n \delta}$$

------------ (15)

Subtracting 13 & 12 gives

$$S_n = p_n \{\cosh \quad \delta g_n - \gamma_n \sinh \delta g_n\} e^{p_n \delta} \text{ .. (16)}$$

Now substituting 15 &16 in 14 gives

$$P_n[\cosh \quad \delta g_n \quad \sinh p_n(g-\delta) + \gamma_n$$
$$\sinh \delta g_n \cosh P_n (g - \delta)] = \frac{k_n \mu_0}{2p_n}$$

$$\therefore p_n = \frac{k_n \mu_0}{2 p_n c_n} \quad \text{----------- (17)}$$

Where c_n = *cosh* δg_n *sinh* $p_n(g-\delta)$ +

$_n$ *sinh* δg_n *cosh* $p_n(g-\delta)$

Substituting 17 in 15 &16 we get

$$R_n = \frac{k_n \mu_0}{2 p_n C_n}\{\cosh \delta g_n + \gamma_n \sinh \delta g_n\} e^{-p_n \delta}$$

-------------- (18)

$$S_n = \frac{k_n \mu_0}{2 p_n C_n}\{\cosh \delta g_n - \gamma_n \sinh \delta g_n\} e^{p_n \delta}$$

-------------- (19)

Therefore equation 11 becomes:-

$$Ap_n(Z) = \frac{k_n \mu_0}{p_n c_n} \cosh g_n Z \quad \text{----------} (20)$$

$$Ag_n(Z) = \frac{k_n \mu_0}{p_n c_n}[\cosh \delta g_n \cosh p_n(Z-\delta) + \gamma_n \sinh \delta g_n \sinh p_n(Z-\delta)]$$

------ (21)

The braking force of the nth harmonic is given by the following Equations:

$$Fb_n = \int_{-\tau/2}^{\tau/2} \int_0^{\delta} \mathrm{Re}(-\sigma \frac{\partial Ap_n}{\partial t} \cdot \frac{\partial Ap_{n^*}}{\partial x}) dxdz \quad \text{------} (22)$$

$$Ap_n(x,z,t) = Ap_n(z) \quad e^{jp(nx+vt)}$$

i.e $$\frac{\partial Ap_n}{\partial t} = jpv \quad Ap_n(z) e^{jp(nx+vt)} \quad \text{------} (23)$$

$$\frac{\partial Ap_{n^*}}{\partial x} = -jp_n \quad Ap_n(z) e^{-jp(nx+vt)} \quad \text{------} (24)$$

After substituting Equation 23&24 in 22 and integrating, the total solution of Equation 22 can be written as: -

$$Fb_n = \frac{-\sigma V \tau k^2{}_n \mu_0^2}{4n|C_n|^2} \left[\frac{\sinh 2\alpha_n \delta}{\alpha_n} + \frac{\sin 2\beta_n \delta}{\beta_n} \right]$$

-------------- (25)

Where

$$C_n = \cosh \delta g_n \sinh p_n(g-\delta) + \gamma_n \sinh \delta g_n \cosh p_n(g-\delta)$$

$$\alpha_n = n^2 p^2$$

$$\beta_n = \mu \sigma VP$$

The fundamental component of braking force for n=1 is:

$$Fb_1 = \frac{-\sigma V \tau k^2{}_1 \mu_0^2}{4|C_1|^2} \left[\frac{\sinh 2\alpha_1 \delta}{\alpha_1} + \frac{\sin 2\beta_1 \delta}{\beta_1} \right]$$

---------------- (26)

The appropriate solution of equation 25 becomes:

$$Fb = \frac{-\sigma V \tau \mu_0^2}{4} \sum_{n=1}^{\infty} \frac{k_n{}^2}{n|C_n|^2} \left[\frac{\sinh 2\alpha_n \delta}{\alpha_n} + \frac{\sinh 2\beta_n \delta}{\beta_n} \right]$$

Where: $K_n = \frac{4k_0}{n\tau} \left[\sin \frac{n\tau}{2} \sin \frac{n\tau'}{2} \right]$

DISCUSSION OF RESULTS

The calculation of the braking force and the effect of the harmonic order at different speed are shown on Figure (3). The baking force Fb against the harmonic order n at speed N=100, 500, 1000, 1500& 2000 r.p.m is shown. It is seen that the braking force increases as higher harmonics are employed, but it is observed that no appreciable increase in the braking force is obtained when the order of harmonic exceeds nine. Therefore it is decided to stop calculations up to the ninth harmonic order.

Figure (4) shows the effect of speed on the braking force at order of harmonic nine. As the speed increases the braking force also increases, but not with the same magnitude, so the curve has a parabolic shape. The braking force will increases as the speed increases, because the disc of the motor at high speed will cuts more number of magnetic lines so the induced emf will be more and the current will be more also, therefore the braking force will be more [12].

CONCLUSIONS

In this investigation, analysis of the problem of eddy current brake is presented which is applicable to brake either thin or thick discs.

A new expression of the braking force is developed which considers the actual width of the pole. This expression is applicable to thin and thick discs covering wide range of working speed. The expression was obtained by solving the two-dimensional field problem of multi regions of different permeability.

A computer program is considered with the help of the flow-chart shown in Figure (5) to compute the braking force and to study the effect of the harmonic order at different speeds. It is found that the results converge rapidly with the harmonic order. No significant change in the results was achieved after the ninth harmonic.

REFERENCES

[1]David Halliday, Robert Resnick, and Jearl Walker, " Fundamentals of Physics, " 5th ed. John Wiley & Sons, New York (1997).

[2] Davis, L. C. and Reitz, J. R. " Eddy Currents in finite conducting Sheets," Journal of Applied Physics, Vol. 42 , No. 11, (1971) .

[3] Schieber, D. , " Braking Torque on Rotating Sheet in Stationary Magnetic Field,"Proceedings of the Institution of Electrical Engineers,Vol.121, No. 2, (1974).

[4] Schieber, D. , " Optimal Dimensions of Rectangular Electromagnet for Braking Purpose," IEEE Transaction on Magnetics, Vol. 11, No. 3, (1975).

[5] Venkatanatnam, K.and Ramachandra, D.," Analysis of eddy current brakes with non magnetic rotors", Proc. IEE, Vol. 124 , No. 1, (1977).

[6] Nagaya, K., Kojima, H., Karube, Y., and Kibayashi, "Braking Force and Damping Coefficient of Eddy Current Brakes Consisting of Cylindrical Magnets and Plate Conductors of Arbitrary Shape." IEEE Transactions on Magnetics, Vol. 20, No. 6, (1984).

[7] Wiederick, H. D., Gauthier, N., Campbell, D. A., and Rochon, P., " Magnetic Braking : Simple Theory and

Experiment," American Journal of Physics, Vol. 55, No. 6, (1987).

[8] Cadwell, L. H., " Magnetic Damping: Analysis of an Eddy Current Brake Using an Air Track," Journal of Physics, Vol. 64, (1996).

[9] Lee, K. and Park, K., " Optimal Robust Control of a Contact-less Brake System Using an Eddy Current," Mechatronics , Vol. 9, No. 6, (1999).

[10] Lee, K. and Park, K., "Modeling of the Eddy Currents with the Consideration of the Induced Magnetic Flux,"Proceedings of the IEEE International Conference, (2001a).

[11] Lee, K. and Park, K. ," Modeling Eddy Current with Boundary Condition by Using Coulomb's Law and the Method of Images," IEEE Transactions on Magnetic.Vol. 38, No. 2 , (2002).

[12]Nguyen-Dai,Quoe-Cuong ,"Contactless Braking System Using Electromagnetism" Part 4 Project 2004.

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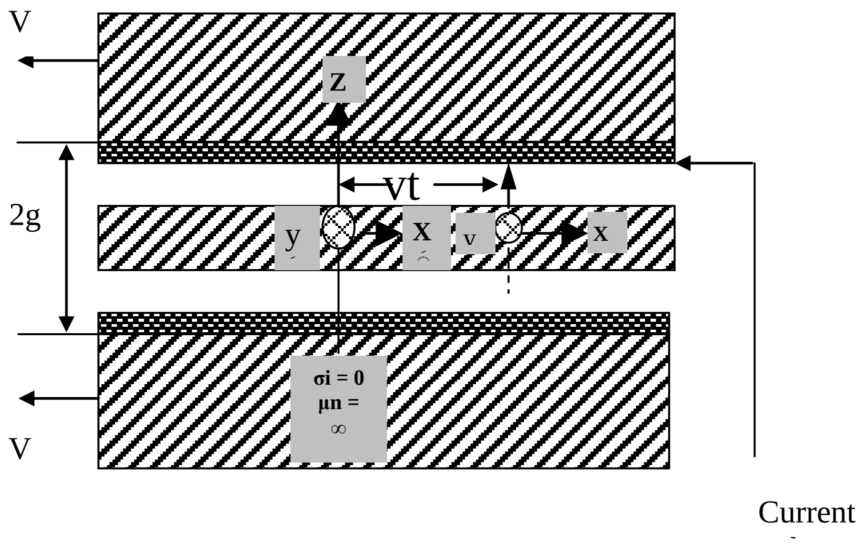

Fig. (1): One dimensional quasi – static brake model used for analysis

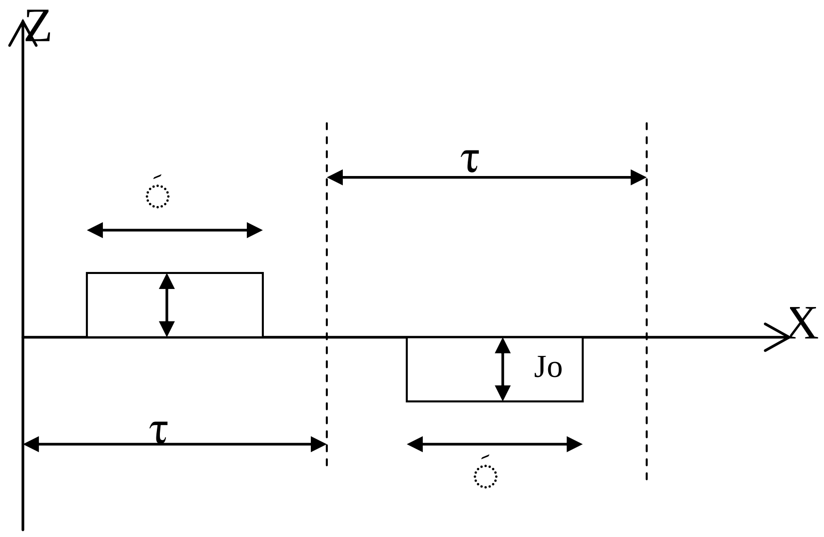

Fig.(2): Repeatable step current distribution

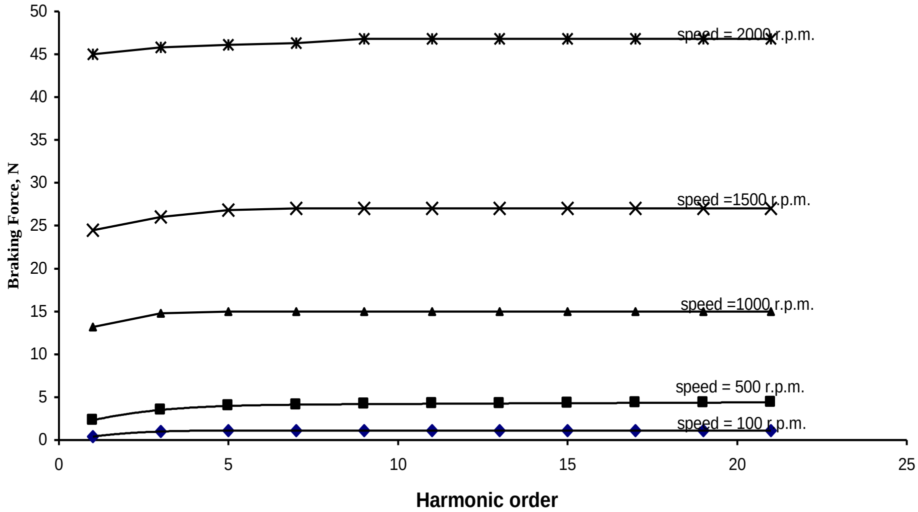

Fig.(3) Effect of harmonic order on the braking force at different speed

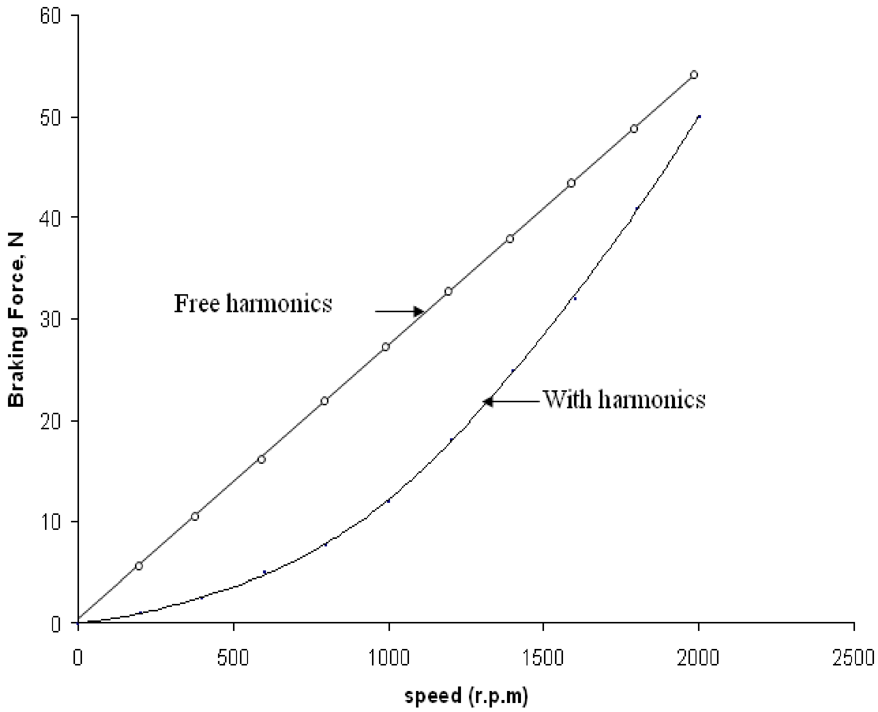

Fig (4) Effect of speed on the braking force at harmonic order nine

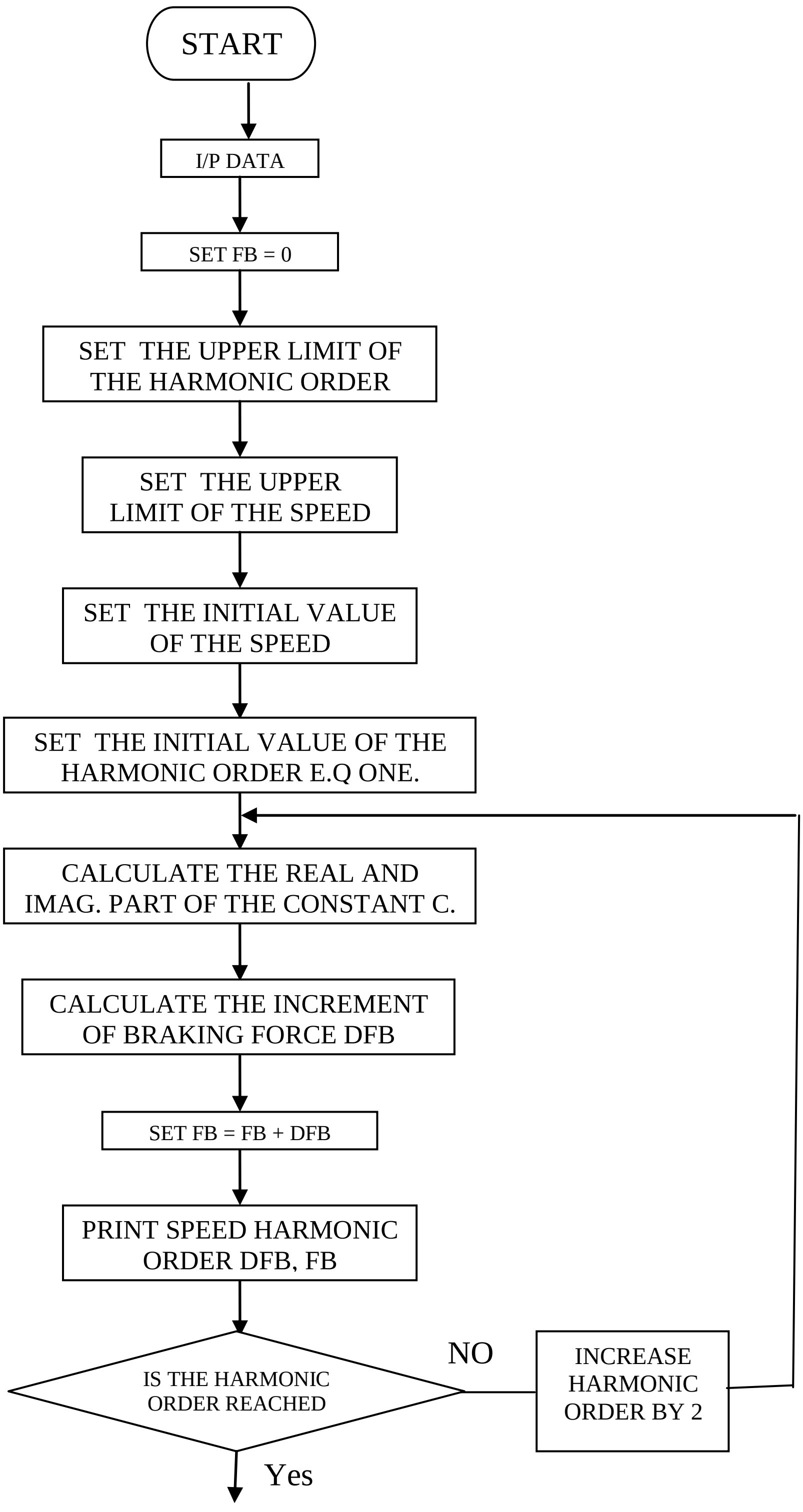

Figure (5) flow chart

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تحليل رتبة التوافقية المؤثرة على قوة كبح التيار ألدوامي في المحركات الكهربائية

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الخلاصة

يهدف البحث إلى تحليل كبح التيار ألدوامي في المحركات الكهربائية باستخدام الحاسوب ،والتي تمثل صيغة لقوة الكبح عند الاخذ بنظر الاعتبار عرض حقيقي للقطب.هذه الصيغة تكون مناسبة عند استخدام دوار سميك أو رقيق ضمن مدى واسع من السرع التي تدور بها المحركات الكهربائية.ولهذا الغرض تم تحضير تحليل رياضي للمسالة مع صيغة قوة الكبح في آن واحد . إذ تم تمثيل الكبح أولا بنموذج رياضي يعتمد على عدد من الفرضيات ثم الحصول على قوة الكبح كنتيجة لحل مسالة المجال حيث تم تبسيط المسالة الى مسالة احادية البعد والحصول على متجه الجهد المغناطيسي ومن ثم الحصول على رتبة التوافقية ذات الترتيب n لقوة كبح التيار ألدوامي باستخدام معادلة قوة لورنتس.

الكلمات الدالة
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