

## **THE OPTIMIZATION OF MACHINING OPERATIONS BASED ON NON-QUADRATIC MODEL VIA SUMT**

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### **ABSTRACT**

The need for higher production rates and lower production cost emphasized the need to increase the rate of metal removal on metal cutting machine tools. This is however, limited by the different technical and technological constraints of the machining process so that the problem has to be handled as an optimization to determining the optimum values of the machining parameters. Several optimization techniques have been developed to solve this problem such as linear and non linear programming methods. However, due to the complexity of the mathematical models and the relatively large number of constraints involved, there is no obvious solution.

This paper introduces a developed algorithm and compared to one of the well known iteration method that is the Sequential Unconstrained Minimization Technique (SUMT). The comparison through an example problem shows that the developed algorithm is more efficient, accurate, as well as quicker the optimum solution.

### **KEYWORDS**

SUMT, Algorithm Performance, NOF, NOI.

## NOTATIONS

- $n$  : Dimension of the problem;  
 $m$ : Number of constraints;  
 $v_L$ : Minimum cutting speed, m/min;  
 $v_u$ : Maximum cutting speed, m/min;  
 $v$ : cutting speed, m/min;  
 $f$ : feed rate, mm/rev;  
 $d$ : depth of cut per pass, mm/pass;  
 $T$ : Tool life, min/edge;  
 $a_i$  : Exponents of ( $f, d, & T$ ) in Taylor's equation;  
 $k$ : Taylor's constant;  
 $t_m$ : Machining time, min;  
 $t_c$ : Tool changing time, min/edge;  
 $D$ : Diameter of work piece, mm;  
 $L$ : Length of work piece, mm;  
 $T_p$  : production time, min/pc;  
 $C_p$  : production cost, \$/pc;  
 $C_o$ : Operating cost, \$/min;  
 $C_t$ : Tool cost, \$/edge;  
 $Hp_{max}$ : Maximum horse power, Kw;  
 $F_{max}$  : Maximum cutting force, N;  
 $SR_{max}$ : Maximum surface roughness,  $\mu m$ ;  
 $\theta_{max}$  : Maximum cutting temperature,  $C^\circ$ ;  
 $\rho$  : Scalar;  
 $g$  :  $n \times 1$  gradient of  $f(x)$  ;  
 $g^*, g^{**}$  : specific  $n \times 1$  gradients of  $f(x)$  ;  
 $i$ :  $i^{th}$  iteration;  
 $H = n \times n$  approximation to Hessian matrix;

$P$ :  $n \times 1$  search direction vector =  $-Hg$ ;

Line search along  $P_i$  giving  $x_{i+1} = x_i + \lambda_i P_i$  ;

$\lambda$ : step size of line search;

$y = g_{i+1} - g_i = n \times 1$  difference vector between two successive gradients;

$S = x_{i+1} - x_i = n \times 1$  difference vector between two successive points;

NOI: Number of iteration evaluations;

NOF: Number of function evaluations;

$Z^+$ : each variable tends to optimal value;

## INTRODUCTION

Machining economy depends on the assigned values of the machining conditions; cutting speed, feed, and depth of cut.

In view of the recent developments of metal cutting machine tools toward automation and the associated high capital costs it is necessary to make full utilization of the machine tools, a significant reduction of the machining time and cost can still be attained by increasing the rate of metal removal. This is however limited by several technical and technological constraints<sup>[1,2,3,4]</sup>.

The assignment of the machining conditions should therefore be based on a techno-economical basis. This means that it required to determine those values of the machining conditions those lead to minimum production cost and at the same time satisfy all the encountered constraints.

Several linear and nonlinear optimization techniques have been employed to obtain the most economic machining conditions. Due to the relatively large number of constraints involved, the computational efficiency of linear programming is considerably reduced<sup>[4,5,6,7]</sup>.

This paper introduces a comparison between some nonlinear optimization methods such as a developed SUMT algorithm<sup>[8]</sup> and the

classical SUMT algorithm<sup>[9]</sup>, and then compare each of them with Nelder-Mead algorithm which was used by another researcher<sup>[6]</sup> in optimization of the constrained machining operations.

## MACHINING ECONOMY

The complexity of this problem is determined by the nature of the objective function and constraints set. In fact only the variable cost per piece will affect the determination of the economic machining conditions. The variable cost consists of some nonlinear functions of (  $v$ ,  $f$  &  $d$  ) with constraints determined by the machining configuration and environment. The function is nonlinear due to the presence of terms for cutting time and tool life.

Typical expressions for a turning operation are:

For machining time

$$t_m = D * L / 1000 * v * f \quad \dots\dots\dots(1)$$

And Taylor's expanded tool life equation is

$$v * f^{a_1} * d^{a_2} * T^{a_3} = K \quad \dots\dots\dots(2)$$

The solution is typically constrained by physical limits on the machining conditions, as well as by restrictions on cutting force, available power, surface roughness, and tool temperature.

Thus two objective functions; production cost and production time are considered in this paper. The production cost can be written as:

$$\begin{aligned} C_p &= C_o * t_m + (C_o * t_c + C_t) * t_m / T \quad \dots\dots\dots(3) \\ &= C_o * A * v^{-1} * f^{-1} + A * v^{(1/a_3-1)} * F^{(a_1/a_3-1)} * D^{(a_1/a_3)} * K^{(-1/a_3)} * (C_o * t_c + C_t) \end{aligned}$$

Where  $A = \pi D * L / 1000$ .

The production time also can be written as:

$$\begin{aligned} T_p &= t_m + t_c - t_m / T \quad \dots\dots\dots(4) \\ &= A * v^{-1} * f^{-1} + t_c * A * v^{(1/a_3-1)} * F^{(a_1/a_3-1)} * D^{(a_2/a_3)} * K^{(-1/a_3)} \end{aligned}$$

These two objective functions can be minimized subjected to the following physical constraints [6].

i- Minimum & maximum permissible machining conditions

$$v_L \leq v \leq v_u, f_L \leq f \leq f_u, \text{ and } d_L \leq d \leq d_u \quad \dots\dots\dots(5)$$

ii- Power limitation, the power consumption allowed on steel is given as a function of machining conditions

$$0.0373 * v^{.91} * f^{.78} * d^{.75} \leq H_{p \max} \quad \dots\dots\dots(6)$$

iii- Maximum cutting force allowed

$$844 * v^{-.1013} * f^{.725} * d^{.75} \leq F_{\max} \quad \dots\dots\dots(7)$$

iv- Surface roughness limitation for steel is given by

$$14785 * v^{-1.52} * f^{1.004} * d^{.25} \leq SR_{\max} \quad \dots\dots\dots(8)$$

v- Temperature constraint for the tool-part contact zone is given by

$$74.96 * v^{.4} * f^{.2} * d^{.105} - 17.8 \leq \theta_{\max} \quad \dots\dots\dots(9)$$

## MINIMIZATION ALGORITHM

The proposed SUMT Algorithm is one of the indirect ( i.e. gradient and conjugate gradient) methods. It was applied to the generally successful inverse barrier function method, which is to be minimized, it takes the form<sup>[9]</sup>:

$$\Phi(x, r) = f(x) + r_i \sum_{j=1}^{j=m} \frac{1}{C_j(x)} \quad \dots\dots\dots(10)$$

The defining  $\Phi(x, r)$  becomes infinite at the boundary of the feasible region  $R$ , i.e. barriers are constructed on each constraint, and the solution  $x_{min}(r) \in R$ ; then  $x^*$ , is approached from the interior of  $R$  in a sequence defined by the controlling parameter  $r$ . The inverse barrier function method is only suitable for inequality constraints. The function  $\Phi(x, r)$  is to be minimized by SUMT and the developed SUMT Algorithms. The term  $f(x)$  can be taken in the form of equation (3) for minimization of machining

cost, or in the form of equation (4) for minimization of machining time, while the term  $(\sum 1/C_j(x))$  includes all constraints in equations (5,6,...,9).

Where a sequence of  $r$  values tending to zero is used, then the effect of the barrier term is steadily reduced to take effect nearer to the boundary of the feasible region. The growth of the constraints set  $(C_j^{-1}(x))$  can be controlled or "canceled" by decreasing  $r$ . Each constraint has its inverse barrier function, which has the necessary property that  $C_j^{-1}(x) \rightarrow \infty$  as  $C_j(x) \rightarrow 0$ .

In order to exploit the inverse barrier function method in practice the Algorithm in Figure (1) was considered. The difference between the classical SUMT and the developed SUMT is in value of the scalar  $\rho$ , while the developed SUMT determines it through each iteration by<sup>[8]</sup>:

$$\rho_i = \frac{(g_{i+1/2}^*, g_{i+1}^*)(g_i^{**}, P_i)}{(g_{i+1}^{**}, g_{i+1}^*)(g_{i+1/2}^{**}, P_i)}, \quad \dots\dots\dots(11)$$

the classical SUMT always using  $\rho = 1$ . Each of them uses it in finding  $y_{i+1} = g_{i+1} - g_i / \rho_i$  and then updating H matrix by the form<sup>[7,10]</sup>:

$$H_{i+1} = H_i + \frac{S_i S_i^T}{S_i^T y_i} - \frac{H_i y_i \cdot y_i^T H_i}{y_i^T H_i y_i} \quad \dots\dots\dots(12)$$

The initial value given to  $r$  is important in reducing the *NOI* to minimize  $\Phi(x, r)$ .  $r$  is chose so it is becoming very small at the optimal point.

## COMUTATIONAL EXPERIENCE

Each of classical SUMT and developed SUMT approaches are used to minimize equation (10). The classical SUMT was used with its related  $\rho_i = 1$ , while developed SUMT was used with its related  $\rho_i$  of the form of equation (11). They are compared with respect to accuracy, efficiency and their speed to reach the optimum solution by means of an example.

This example describes a case of turning plain carbon steel with out coolant, in which, both production cost (i.e. eq. 3) and production time (i.e. eq. 4) can be considered in stead of  $f(x)$  in equation (10), while all constraints in equations (5,6,...,9) can be considered in the term  $(\sum 1/C_j(x))$  of such equation.

The conditions of example are given in Table (1). These conditions had been used by Agapiou <sup>[6]</sup>. He uses it with a different approach which is called Nelder- Mead approach, ( it relates to the direct methods<sup>[10]</sup>), in optimization of production cost.

The optimal solutions of production cost which are found by using each of classical SUMT and its development with data in Table (1), for several initial vectors  $(v_0, f_0)$ , are summarized in Table (2). The initial vectors are used by assuming different machining passes each of them affected by corresponding surface roughness constraint and diameter of part. Since  $d$  has the lowest affect on tool life, hence it takes its upper fixed bound as possible <sup>[7]</sup>.

Each of classical SUMT and developed SUMT approaches are compared with Nelder- Mead approach. The comparison results are summarized in Table (3).

## CONCLUSIONS

1. The comparison results in Table (2) shows that the developed SUMT approach is more accurate, efficient, as well as quicker to approach the optimum solution than the classical SUMT approach. It decreases the execution time by 8.1%, production cost by 0.335% per piece, No. of iteration evaluations by 19%, and No. of function evaluations by 19.81% compared with the classical SUMT approach.

2. The comparison results in Table (3) indicates that each of classical SUMT and developed SUMT approaches are more accurate than Nelder- Mead approach. Each of them decreases the production cost per piece by 24% of that of Nelder- Mead approach.

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Table(1) Example conditions

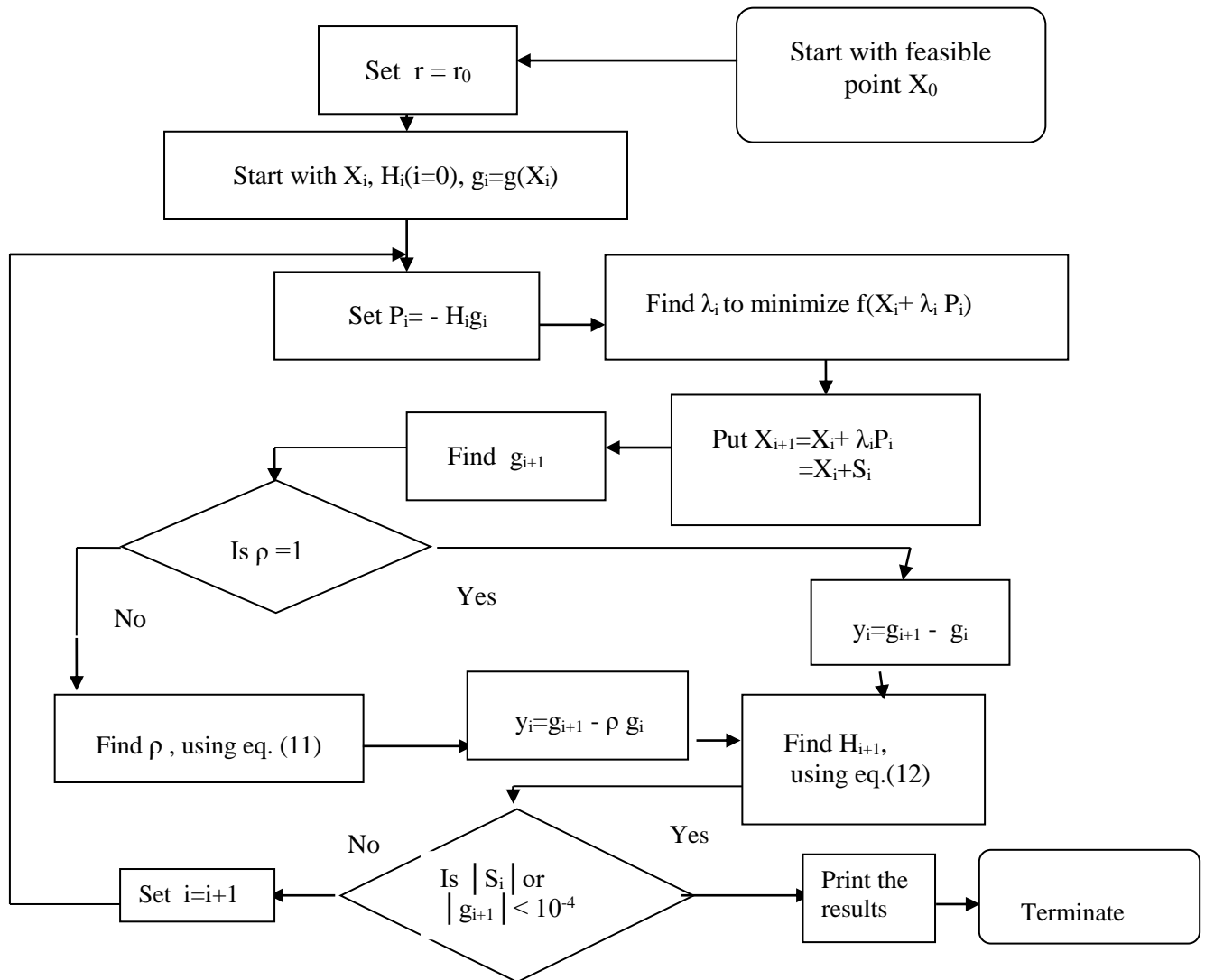
Parameter	Value	Parameter	Value	Parameter	Value
L	203 mm	D	mm, different values	$v_L$	30 m/min.
$v_u$	200 m/min.	$f_L$	0.254 mm/rev.	$f_u$	0.762 mm/rev.
$d_L$	1.2 mm	$d_u$	6.0 mm	$SR_{max}$	8.2 $\mu$ m
$Hp_{max}$	5.0 Kw	$F_{max}$	1100 N	$\theta_{max}$	500 C°
$a_1$	0.29	$a_2$	0.35	$a_3$	0.25
K	193.3	$t_c$	0.5 min/edge	$C_o$	0.1 \$/min
$C_t$	0.5 \$/edge				

Table (2) Comparison of classical SUMT and developed SUM

Pass Corresponding			Classical SUMT approach						Initial Vector		Developed SUMT approach					
Diam. mm	Rough $\mu$ m	Depth. mm	NOI	NOF	Ex.time sec	$C_u^+$	$f^+$	$v^+$	$v_o$	$f_o$	NOI	NOF	Ex.time sec	$C_u^+$	$f^+$	$v^+$
152	8	5.08	33	156	63	.546	.347	91.51	115	.35	27	138	58	.546	.351	92.26
152	8	4.06	28	131	55	.464	.397	96.4	100	.34	25	113	52	.463	.392	95.63
152	8	2.54	29	127	50	.33	.524	107.2	105	.4	22	103	45	.328	.507	104.99
141.84	8	3.81	31	151	60	.413	.404	96.54	110	.4	26	137	58	.413	.406	96.78
134.22	2	1.27	29	139	58	.345	.29	160.7	180	.3	24	93	55	.34	.32	160
133.72	2	1.02	23	124	48	.292	.32	164.6	165	.3	16	80	39	.292	.32	165.6
Total			173	828	334	2.39					140	664	307	2.382		

Table (3) Comparison of classical SUMT and developed SUMT approaches with Nelder –Mead approach

Pass Corresponding			Classical SUMT approach			Nelder-Mead approach			Developed SUMT approach		
Diam. mm	Rough. $\mu$ m	Depth. mm	$C_u^+$	$f^+$	$v^+$	$v_o$	$f_o$	$C_u^+$	$C_u^+$	$f^+$	$v^+$
152	8	5.08	.546	.347	91.51	112	.292	.83	.546	.351	92.26
152	8	4.06	.464	.397	96.4	94	.330	.57	.463	.392	95.63
152	8	2.54	.33	.524	107.2	117	.318	.43	.328	.507	104.99
141.84	8	3.81	.413	.404	96.54	108	.371	.52	.413	.406	96.78
134.22	2	1.27	.345	.29	160.7	173	.297	.44	.34	.32	160
133.72	2	1.02	.292	.32	164.6	178	.337	.34	.292	.32	165.6
Total			2.39					3.13	2.382		
Comparison with r.to Nelder-Mead approach by P.Cost/Pc			Classical SUMT $C_u^+=$ (3.13-2.39)/3.13=23.64%					Developed SUMT $C_u^+=$ (3.13-2.382)/3.13=23.9%			



**Figure (1) The Algorithm Flow chart**

## استخدام خوارزمية جديدة للحل التتابعي لظروف التشغيل الميكانيكي المثلى

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### الخلاصة

يرافق الحاجة إلى معدلات الإنتاج العليا وكلفه الدنيا زيادة في معدلات المعدن المزاح بواسطة آلات القطع. يقيد تلك الزيادة قيود تكنولوجية مختلفة تتحكم بعمليات التشغيل الميكانيكي مما يستوجب التحديد الأمثل لقيم ظروف التشغيل، مما استوجب تطوير تقنيات الامثلية الخطية واللاخطية لحل تلك المشكلة.

على الرغم من التطور الحاصل في هذا المجال فإنه نتيجة لتعقيد النماذج الرياضية والقيود التكنولوجية التي تتحكم بها لم يتم التوصل إلى حل جلي لهذه المشكلة. يتعامل هذا البحث مع تطوير لتقنية التقليل التتابعي اللامقيد (SUMT) واستخدامها في تقليل المسائل المقيدة اللاخطية. يتم مقارنة الخوارزمية المطورة مع الخوارزمية الأصلية من خلال مثال في الخراطة السطحية. أظهرت المقارنة كفاءة الخوارزمية المطورة من ناحية الدقة والسرعة في الوصول إلى الحل الأمثل.

### الكلمات الدالة

المسائل المقيدة، كفاءة الخوارزميات، تقنية التقليل المتتابع اللامقيد