NONLINEAR DYNAMIC ANALYSIS OF REINFORCED CONCRETE SLABS

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ABSTRACT

In this study the nonlinear transient dynamic analysis of reinforced concrete slabs using the finite element method is presented. Eight-node Serendipity degenerated elements have been employed. This element is based on isoparametric principles with modifications, which relax excessive constraints. The modifications include reduced order integration to overcome the shear locking.

A layered approach is adopted to discretize the concrete through the thickness. Both an elastic-perfectly plastic and strain hardening plasticity approaches have been employed to model the compressive behavior of the concrete. A tensile strength criterion is used to initiation of crack and a smeared fixed crack approach is used to model the behavior of the cracked concrete. Tension stiffening in concrete is assumed such that the concrete can take some tension after cracking.

Implicit Newmark with corrector-predictor algorithm is employed for time integration of the equation of motion.

Several examples are analyzed using the proposed model. The numerical results showed good agreement with other sources.

KEY WORDS

Dynamics, Finite elements, Nonlinear analysis, Reduced integration, Reinforced concrete slabs.

NOTATIONS

B Strain-nodal displacement matrix.

B_b Bending strain- nodal displacement matrix.

B_s Transverse shear strain- nodal displacement matrix.

D Flexural (or Shear) Rigidities.

D_b Flexural rigidities.

D_s Shear rigidities.

d Displacements.

$\dot{d}$ Velocities.

$\ddot{d}$ Accelerations

E_c Initial modulus of elasticity of concrete.

E_s Modulus of elasticity of steel.

E_s' Second modulus of elasticity of steel (hardening coefficient).

f_c' Uniaxial compressive strength of concrete.

f_t' Uniaxial tensile strength of concrete.

Gc Fracture energy of concrete.

K Elastic stiffness matrix.

K^{*} Effective stiffness matrix.

K_T Tangential stiffness matrix.

M_x, M_y, M_{xy} Generalized stress components(moments).

N Shape function.

ρ Mass density.

Q_x, Q_y Generalized stress components (shear forces).

R.C. Reinforced concrete.

β, γ Newmark's integration parameters.

ε_x, ε_y Strains in x and y-direction.

ε_b Bending strain tensor.

ε_s Transverse shear strain tensor.

ε_u Crushing strain.

ν Poisson's ratio.

σ_x, σ_y Normal stress components.

INTRODUCTION

The finite element method was introduced for structural analysis many years ago. It has been recognized as a powerful and widely used approach for analysis of R.C. plates and shells.

Beshara and Virdi[1] analyzed plane and axisymmetric reinforced concrete structures subjected to transient impulsive loading. A strain rate sensitive elasto-viscoplastic model is developed for the compressive behavior of concrete. Steel is modeled as a strain rate sensitive uniaxial elasto-viscoplastic material with linear hardening.

Hinton[2] analyzed reinforced concrete plates and shells under transient dynamic loading. The three dimensional isoparametric element with 20 nodes is used to simulate the concrete. A viscoplastic model is used to simulate the concrete in compression with two surfaces, the failure surface which indicates the initiation of degradation of material and the yield surface which indicates the initiation of yielding state. Cracking and crushing of concrete are taken into account. The nonlinear behavior of steel is simulated by a simple viscoplastic formula.

Lopez Cela et al[3] analyzed a thin reinforced concrete shell subjected to impact load. The Drucker-Prager elastic-plastic criterion was used for concrete and viscoplastic regularization technique was applied in order to prevent appearance of unphysical strain localizations.

Riera and Iturrioz[4] analyzed reinforced concrete plates and shells subjected to impulsive loading. Discrete element models were used for evaluating impact and impulsive response of reinforced concrete plates and shells. Yielding of steel as well as fracture of concrete was duly accounted for by means of a constitutive criterion that quantifies coupling between both effects.

Shirai et al [5] investigated and proposed a method to improve impact resistance of reinforced concrete plates against projectile impact, and the damage of double-layered reinforced concrete plates was examined experimentally and simulated analytically.

Sziveri et al [6] analyzed reinforced concrete plates under transient dynamic loading. A layered triangular element was considered for determining the dynamic transient nonlinear response of reinforced concrete plates. In this study, steel was modeled as a strain rate sensitive uniaxial elasto-viscoplastic material to account for the strain rate sensitivity as well as stress-strain dependence. For the behavior of concrete, a strain rate sensitive hardening-softening elasto-viscoplastic model with two rate dependent surface was utilized. In tensile region, the concrete was modeled as a linear elastic strain softening material in which the crack initiation was determined by a proposed strain rate criterion. The cracks were smeared in the concrete elements.

BASIC THEORY

By Mindlin thick plate element, the variation of displacements and rotations are given by the expression as[7]:

$$[w,\ \theta_x,\ \theta_y\ ]^T = \sum_{i=1}^{n} N_i d_i \qquad \text{......................................(1)}$$

The plate curvature-displacement and shear strain-displacement relations are then written as:

$$\varepsilon_b = \sum_{i=1}^{n} B_{bi} d_i \quad , \quad \varepsilon_s = \sum_{i=1}^{n} B_{si} d_i \qquad \text{.................................(2)}$$

The moment-curvature and shear force–shear strain relations can be written as:

$$[M_x,\ M_y,\ M_{xy}\ ]^T = D_b\varepsilon_b \quad , \quad [Q_x,\ Q_y\ ]^T = D_s\varepsilon_s \quad .....(3)$$

Based on the energy minimization, the elastic stiffness and the mass matrices can be determined from the relations:

$$[K]=\int_v [B]^T [D][B]dv \quad ....(5)$$

$$[M]=\rho\int_v [N]^T [N]dv \quad ....(6)$$

REDUCED INTEGRATION

Based on the work of Dohestry et al[8] to eliminate the parasitic shear on plane quadrilateral elements, the implementation of the reduced integration for the degenerated shell element was firstly introduced by Zeinkiewics et al [9]. Then many papers about the reduced integration technique have been published[10,11,12].

Using the full integration rule a shear-locking problem will appear. Therefore reduced integration rule (2x2) for 8-node Serendibity element is applied in this study to overcome this problem.

MATERIAL MODELING

Based on the flow theory of plasticity, the nonlinear compressive behavior of concrete is modeled. Adopting Kupfer's results[13], the yield condition for the slab can be written in term of the stress components as[14]:

$$f(\sigma) = \left\{1.355\left[(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y) + 3(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)\right] + 0.355\sigma_o(\sigma_x + \sigma_y)\right\}^{0.5} = \sigma_o \quad \text{......(7)}$$

where (σ_o) is the equivalent effective stress taken as the compressive strength ($f_c^{'}$) which is obtained from uniaxial test.

Both perfectly plastic and strain hardening models are represented in one-dimensional form in Fig.(1).

The crushing of concrete is a strain control phenomenon. A simple way of incorporation in the model is to convert the yield criterion of stresses directly into the strains, and the crushing condition can be expressed in terms of the total strain components as:

$$1.355\left\{(\varepsilon_x^2 + \varepsilon_y^2 - \varepsilon_x\varepsilon_y) + 0.75\,(\gamma_{xy}^2 + \gamma_{xz}^2 + \gamma_{yz}^2)\right\} + 0.355\varepsilon_u(\varepsilon_x + \varepsilon_y) = \varepsilon_u^2 \quad \text{....(8)}$$

The concrete is assumed to lose all its characteristics of strength and rigidity when (ε_u) reaches the specified ultimate strain.

The response of concrete in tension is assumed to be linearly elastic until the fracture surface is reached. Cracks are assumed to form in planes perpendicular to the direction of the maximum principal tensile stress if this maximum stress reaches the specified concrete tensile strength. After cracking has occurred, a gradual release of the concrete tensile stress component normal to the cracked plane is adopted according to a tension stiffening diagram illustrated in Fig.(2).

Steel reinforcement behavior in tension and compression is modeled by considering the steel bars as layers of equivalent thickness. Each steel layer exhibits uniaxial response, having strength and stiffness characteristics in the bar direction. A bilinear idealization is adopted in order to model the elasto-plastic stress-strain relationships.

Cracked Shear Modulus

The crack width, aggregate size, reinforcement ratio and bar size, are the primary variables in the shear transfer mechanism as indicated in experimental work. The amount of shear stress can be transferred across the rough surfaces of a cracked concrete, and the dowel action of steel is contributing to the shear stiffness across the cracks[15]. An appropriate value of the cracked shear modulus (G) can be estimated in a smeared cracking model[15,16]. In the present study, the cracked shear modulus is assumed to be a function of the current tensile strain. In this approach a value of $(G^{'})$ linearly decreasing with the current tensile strain is adopted by Cedolin and Deipoli[16] and used by many investigators[14,17] .

For concrete cracked in direction 1.

$$G_{12}^{'} = 0.25G(1-\varepsilon_1 / 0.004) \qquad \text{for } \varepsilon_1 < 0.004$$

$$G_{12}^{'} = 0 \qquad \text{for } \varepsilon_1 > 0.004$$

$$G'_{13} = G'_{12}$$
$$G'_{23} = \frac{5}{6}G \qquad \text{..................................(9)}$$

where (G) is the uncracked shear modulus and (ε_1) is the tensile strain in direction (1). For concrete cracked in both directions:

$$G'_{13} = 0.25G(1-\varepsilon_1 / 0.004) \qquad \text{for } \varepsilon_1 < 0.004$$

$$G'_{13} = 0 \qquad \text{for } \varepsilon_1 > 0.004$$

$$G'_{23} = 0.25G(1-\varepsilon_2 / 0.004) \qquad \text{for } \varepsilon_2 < 0.004$$

$$G'_{23} = 0 \qquad \text{for } \varepsilon_2 > 0.004$$

$$G'_{12} = 0.5G'_{23} \qquad \text{for } G'_{23} < G'_{13} \qquad \text{...........(10)}$$

NEWMARK METHOD

The Newmark method as cited in reference[18], and adopted in this work, is an extension of the linear acceleration method. The dynamic equilibrium equation is linearized and written at time t_{n+1} as:

$$M\ddot{d}_{n+1} + C\dot{d}_{n+1} + Kd_{n+1} = f_{n+1} \qquad \text{.......................... (11)}$$

And $[C] = c\int_v [N]^T [N] dv$ ....(12)

where c is a damping coefficient (per unit volume).

The following assumptions on the variation of displacements and velocities are made within a typical time step:

$$d_{n+1} = d_n + \Delta t \dot{d}_n + \frac{\Delta t^2}{2}[(1-2\beta)\ddot{d}_n + 2\ \beta\ \ddot{d}_{n+1}] \quad \text{..............(13)}$$

$$\dot{d}_{n+1} = \dot{d}_n + \Delta t[(1-\gamma)\ddot{d}_n + \gamma\ \ddot{d}_{n+1}] \quad \text{.......................(14)}$$

The Newmark family of direct integration includes, as particular cases, many well known integration schemes.

In the present work an unconditionally stable time stepping scheme is adopted with $\gamma = 0.5$ and $\beta = 0.25$

Huang[7] and Hughes et al[10] have developed a predictor-corrector form of the Newmark method which is most suitable for nonlinear transient analysis.

The Newmark formulas can be written in terms of predictor and corrector values as

$$d_{n+1} = d^p_{n+1} + \Delta t^2 \beta\ \ddot{d}_{n+1} \quad \text{...................................(15)}$$

$$\dot{d}_{n+1} = \dot{d}^p_{n+1} + \Delta t\ \gamma\ \ddot{d}_{n+1} \quad \text{....................................(16)}$$

with predictor values given as

$$d^p{}_{n+1} = d_n + \Delta t \dot{d}_n + \frac{\Delta t^2}{2}(1-2\beta)\ddot{d}_n \quad \text{.........................(17)}$$

$$\dot{d}^p{}_{n+1} = \dot{d}_n + \Delta t(1-\gamma)\ddot{d}_n \quad \text{..............................(18)}$$

The terms d_{n+1} , $\dot{d}_{n+1}$ are corrector values and $d^p{}_{n+1}$, $\dot{d}^p{}_{n+1}$ are the predictor values. The corrector values for the acceleration values can be obtained from equations (15) and (16) as:

$$\ddot{d}_{n+1} = (d_{n+1} - d^{p}_{n+1})/(\beta \Delta t^{2}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Substituting equations(15)and (19) into equation(11) an effective static problem is formed in terms of unknown Δd where:

$$K^{*}\Delta d = \psi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

and where the effective stiffness matrix is

$$K^{*} = M/(\beta\ \Delta t^{2}) + \gamma\ C_{T}/(\beta\ \Delta t) + K_{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

and the residual forces are

$$\psi = f_{n+1} - M\ddot{d}^{p}_{n+1} - C_{T}\dot{d}^{p}_{n+1} - p(d^{p}_{n+1}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Where

$$p(d) = Kd \quad . \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

When solving nonlinear problems, the linearization makes it necessary to perform iterative correction to Δd to achieve equilibrium at time $t+\Delta t$. A Newton-Raphson type scheme is used in this work[9,14,15].

NUMERICAL EXAMPLES

Example(1): Simply supported R.C. beam under step load

A simply supported reinforced concrete beam subjected to two symmetrically applied loads is considered to check the proposed computational method (see Fig.(3)). The material properties are given in Table(1).

Using symmetry condition only one-half of the beam needs to be considered. The elements used and the mesh can be seen in Fig.(4).

Six concrete layers and one steel layer are used in the thickness direction.

The time step is .0005 second (as 1/40 of the elastic fundamental period).

The dynamic response is shown in Figs.(5-6). Numerical results are in excellent agreement with those references[1,2]

Example(2): Clamped circular R.C. slab

The clamped reinforced concrete slab shown in Fig.(7) is subjected to a uniformly distributed load of intensity 0.14 N/mm^2. The slab has a radius of 10m and a thickness of 1m. The load is applied with a rise time equal to half of the elastic fundamental period (T=0.06 second). The materials properties are given in Table (2).

The percentage of reinforcement placed near the upper and lower surfaces in the radial and tangential directions is 1% .

From symmetry only one quarter of the slab is considered. The finite element mesh is shown in Fig.(8). Seven elements with six concrete layers and four steel layers are used in the thickness direction.

The selected time step is approximately 1/100 of the elastic fundamental period this is nearly 0.0005 second.

The dynamic response for different crack strains are shown in Figs.(9-11). Numerical results are in excellent agreement with reference[2].

CONCLUSIONS

A finite element technique has been used successfully for the nonlinear dynamic analysis of reinforced concrete slabs.

No locking was observed in the results due to adopting the 8-node Serendipity element with reduced integration.

An excellent agreement is found between the present results and other source results throughout the entire structural response. This demonstrates the effectiveness of the proposed element and the solution procedure.

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Table(1) Material properties for simply supported R.C. beam of example(1)

E_c N/mm ^{2}	ν	f_c' N/mm ^{2}	ε_u	cracking strain	ρ N.sec ^{2} /mm ^{4}	E_s N/mm ^{2}	f_y N/mm ^{2}
42059	0.2	25.8	0.0035	0.00015	0.2E-8	206800	303.4

Table(2) Material properties for clamped R.C. circular slab of example(2)

E_c N/mm ^{2}	ν	f_c' N/mm ^{2}	ε_u	cracking strain	ρ N.sec ^{2} /mm ^{4}	E_s N/mm ^{2}	f_y N/mm ^{2}
200000	0.2	35.0	0.0035	0.00015 to 0.0002	0.245E-8	210000	460.0

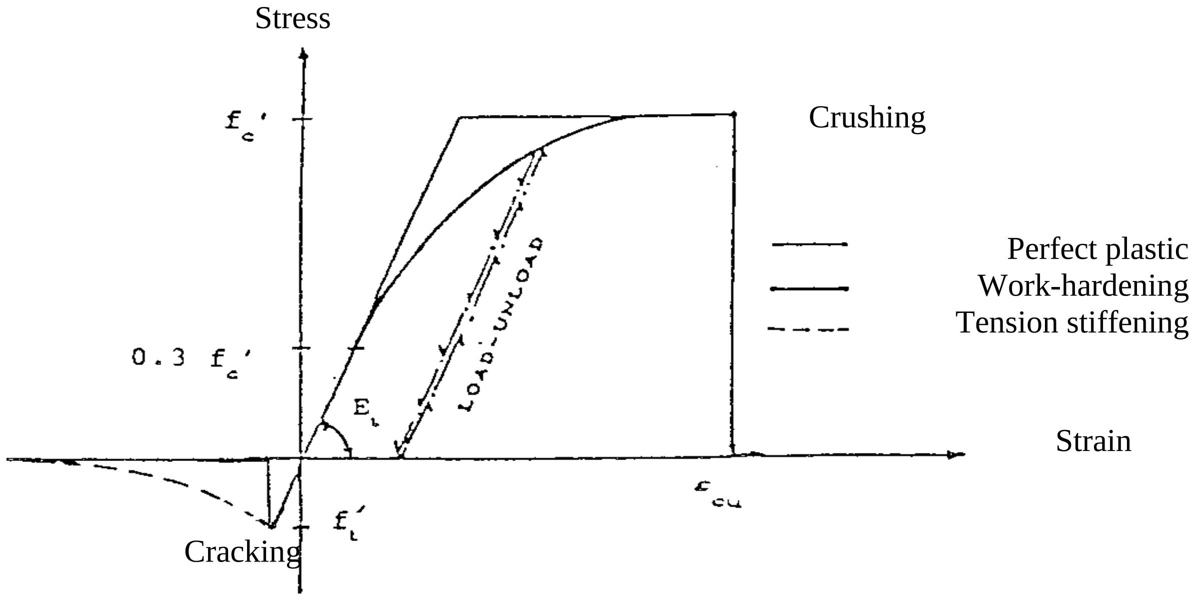

Fig. (1) One-dimensional representation of the concrete constitutive model[14].

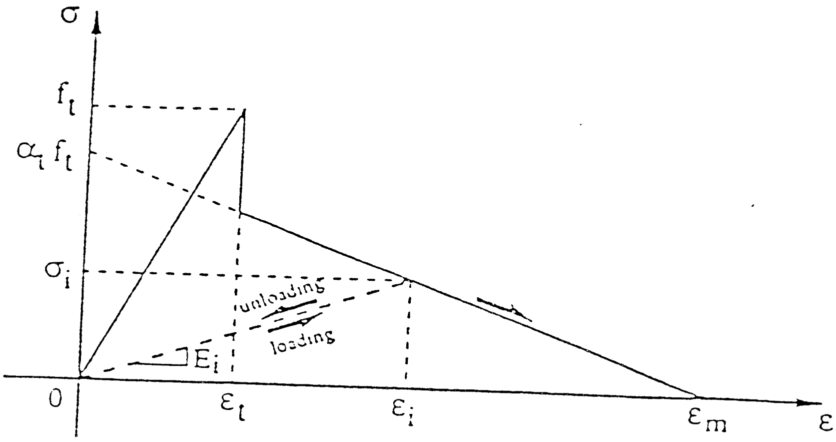

Fig. (2) Loading and unloading behavior of cracked concrete illustrating tension stiffening behavior[14].

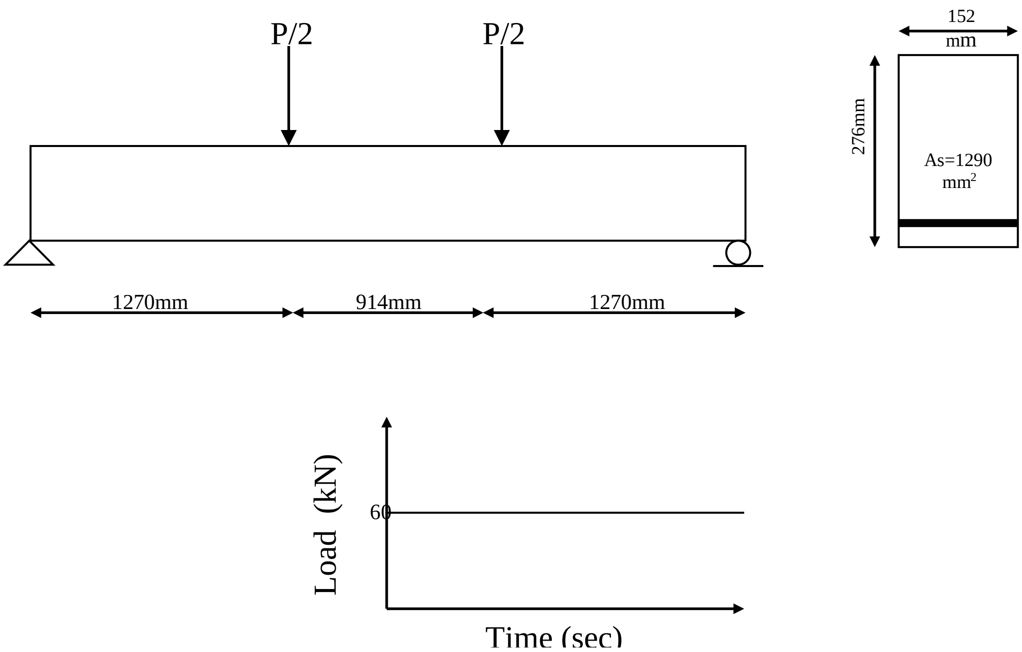

Fig.(3) Geometry, loading and reinforcement details for R.C. beam under step load.

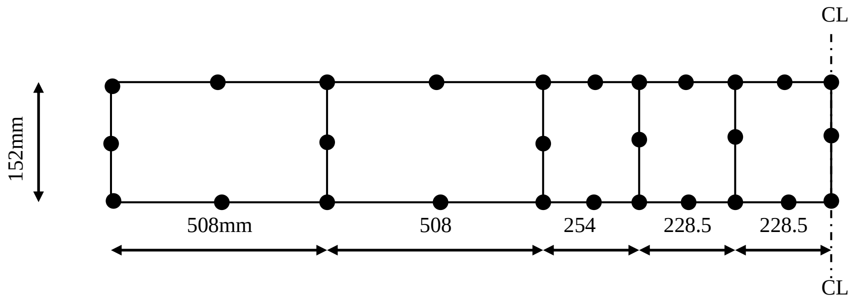

Fig.(4) Finite element mesh for R.C. beam under step load.

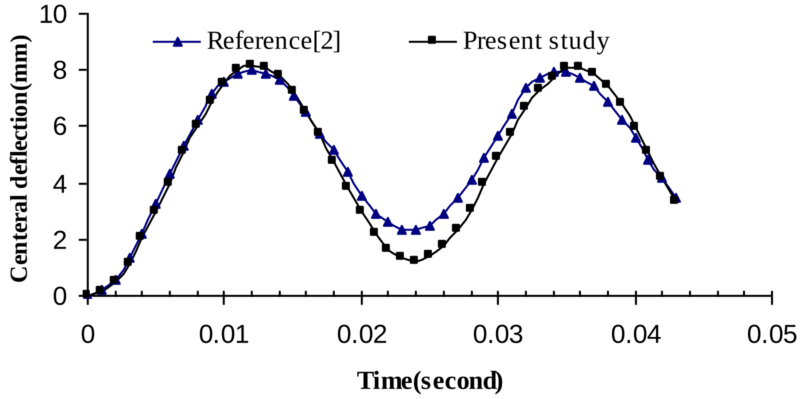

Fig.(5) Nonlinear dynamic response of R.C. beam (elasto-plastic model)

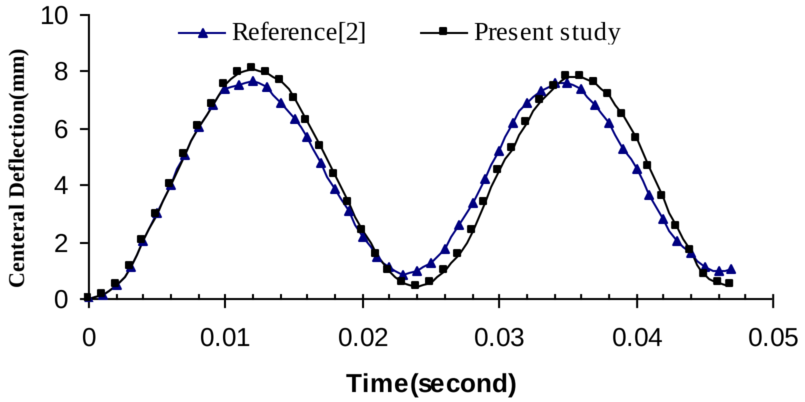

Fig.(6) Nonlinear dynamic response of R.C. beam (strain hardening model)

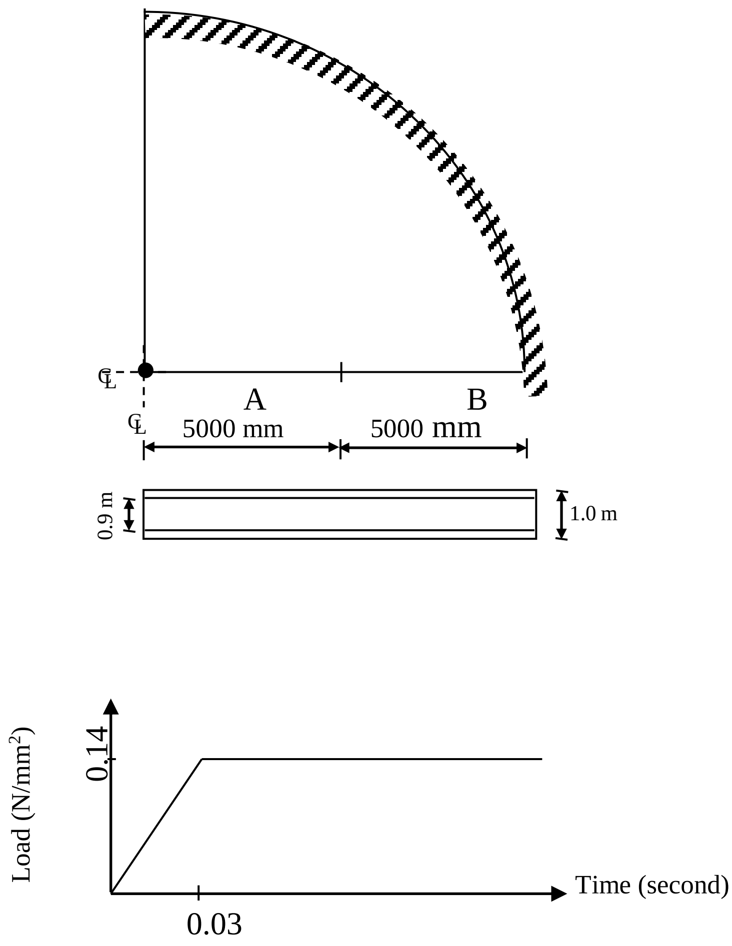

Fig. (7) Geometry and load-time history for example(2)

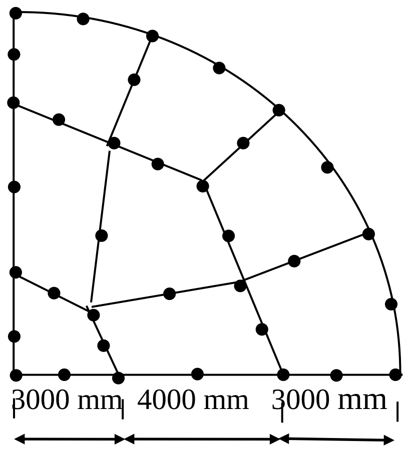

Fig.(8) Finite element mesh for example(2)

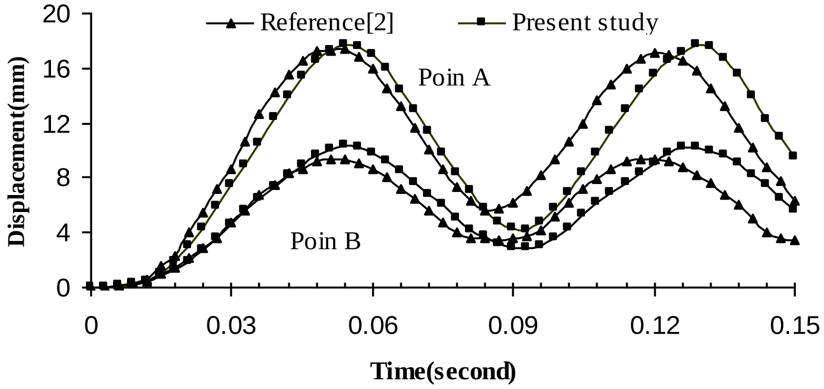

Fig.(9) Nonlinear dynamic response of R.C. circular slab (crack strain=0.00015)

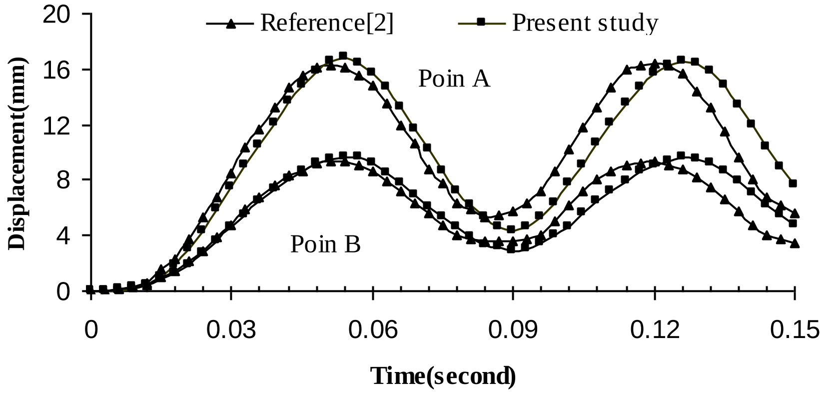

Fig.(10) Nonlinear dynamic response of R.C. circular slab (crack strain=0.00018)

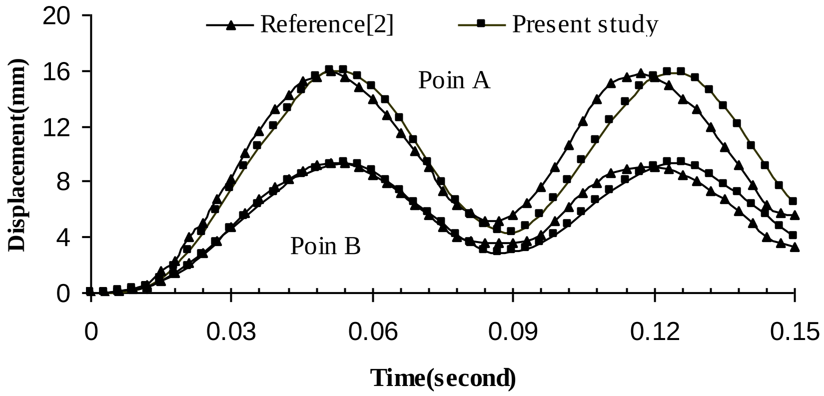

Fig.(11) Nonlinear dynamic response of R.C. circular slab (crack strain =0.0002)

التحليل اللاخطي الديناميكي للبلاطات الخرسانية المسلحة

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الخلاصة

استخدمت طريقة العناصر المحددة لدراسة التصرف اللاخطي الديناميكي للبلاطات الخرسانية المسلحة. استخدمت العناصر ثمانية العقد في هذا البحث. هذه العناصر تعتمد على مبدأ توحيد المتغيرات مع اعتماد مجموعة من التعديلات التي تخفف القيود الإضافية. هذه التعديلات تتضمن قواعد التكامل المخفض وذلك لتفادي حالة القفل بالقص.

استخدم مبدأ الطبقات لتمثيل الخرسانة وحديد التسليح خلال سمك السقف. و مثل سلوك الخرسانة في حالة الانضغاط كمادة مرنة- تامة اللدونة أو كمادة مرنة مع انفعالات لدنة متصلدة. تم استخدام سلوك مقاومة الخرسانة للشد للتنبؤ بحدوث تشقق واستخدم اسلوب الشق الثابت لتمثيل الخرسانة المتشققة مع تثبيت حدود مقاومة تصلد الشد للتنبؤ بحدوث الشق.

تم أعتماد طريقة نيومارك الضمنية مع طريقة التنبؤ-التصحيح لحل معادلة الحركة التفاضلية. تم حل عدة أمثلة أظهرت النتائج توافقا" جيدا" مع النتائج المستحصلة بالطرق الأخرى.