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USING HYDRAULIC PARAMETERS TO ESTIMATE LONGITUDINAL DISPERSION COEFFICIENT IN OPEN CHANNEL

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ABSTRACT

A comparative analysis of previous theoretical and empirical equations is applied to evaluate their behavior in predicting dispersion coefficient in open channel. A new simplified method for predicting dispersion coefficients using hydraulic parameters is developed. A nonlinear multiple regression method is prepared to derive a new equation of dispersion coefficient. This equation is proven to be superior in explaining dispersion coefficient of open channel more precisely, as compared to existing equations.

Keywords: Longitudinal dispersion, hydraulic parameters, dispersion coefficients

LIST OF SYMBOLS

D	Longitudinal dispersion coefficient, (m^2/s)
h	Depth of water, (m).
Q	Flow of water, (m^3/s)
u	Velocity of flow, (m/s)
u_*	Shear velocity, (m/s)
g	Gravitational acceleration , m^2/sec
W	Width of the channel, m
S	Slope of the channel
R	Hydraulic radius, m
D_p	Predicted dispersion coefficient, m^2/sec
D_m	Measured dispersion coefficient, m^2/sec
Re	Reynolds number
ρ	Fluid density, kg/m^3
μ	Fluid viscosity, Pa.sec

INTRODUCTION

Engineers may encounter the problem of predicting the result of accidental spillages of radioactive material (terrorism) or setting the level of discharges from a pollutant source. Whatever the specific application, there is a need for reliable models of radioactivity solute transport in open channel flows and to calculate doses that response from these concentrations. In solute transport models, the velocity and dispersion coefficients in the channel must be known. Dispersion coefficients represent all the mixing processes in the flow [1]. Longitudinal dispersion coefficient can be estimated using the groups of fluid properties; including fluid density, viscosity and hydraulic characteristics; which include mean velocity, shear velocity and depth of flow.

Several empirical and analytical equations for computing the longitudinal dispersion coefficient have been recommended by various investigators. These equations produce values of longitudinal dispersion coefficient which vary widely for the same flow conditions. In this study the dispersion coefficient in the flume of hydraulic laboratory of the Engineering at AL-Mustansiriya University is estimated by measuring the other elements of the mixing process. Several new data have been generated during this work. Twenty four data sets have been used to develop equation which predicts longitudinal dispersion coefficient in open channel using hydraulic parameters.

THEORETICAL AND EMPIRICAL APPROACHES

Taylor [2] first introduced a concept for the longitudinal dispersion coefficient for longitudinal mixing in a straight circular tube in turbulent flow. Taylor derived his equation theoretically as follow:

$$D = 10.11\, U_*.\, r \qquad \text{………(1)}$$

in which r= tube radius; and U_*=shear velocity which is given as

$$U_* = \sqrt{gRS} \qquad \text{…….. (2)}$$

in which g= gravitational acceleration; R=hydraulic radius; and S= the slope of the energy grade line.

Elder [3] extended Taylor method for uniform flow in an open channel of infinite width. He derived a dispersion equation assuming a logarithmic velocity profile and assuming that the mixing coefficient for momentum transfer and mass transfer in the vertical direction are the same. Elder derived the following equation:

$$D = 5.93\, h\, U_* \qquad \text{….. (3)}$$

in which h= depth of flow.

Elder's equation has been widely used because it is simple and has sound theoretical background. However, it has been suggested that his equation may not describe dispersion in natural streams [1]. Fischer [4,5] showed that Elder's equation significantly underestimates the natural dispersion in real streams, because it does not consider the transverse variation of the velocity profile across the stream. He postulated that in most natural streams, the transverse profile of the velocity is far more important than the vertical profile in producing longitudinal dispersion.

Parker [6] adapted Taylor's turbulent flow equation to an open channel by substituting the hydraulic radius for the half pipe radius. The resulting equation is

$$D = 14.28 R^{3/2} \sqrt{2gS} \qquad \text{……(4)}$$

Fischer [7] developed a simpler equation by introducing a reasonable approximation of the triple integration, velocity deviation, and transverse turbulent diffusion coefficient. The result is

$$D = 0.011 \frac{U^2 W^2}{h U_*} \qquad \text{..… (5)}$$

Eq. (5) has the advantage of simplicity in that it can predict dispersion coefficient by using only the data of cross-sectional mean parameters, which are easily obtained for a stream. McQuivey and Keefer [8] developed a simple equation of dispersion coefficient using the similarity between the 1D solute dispersion equation and the 1D flow equation, especially when Froude number is less than 0.5. They initially derived an equation which relates the longitudinal dispersion coefficient and the flow dispersion coefficient. Then by the linear least-square regression of the field data, they derived an empirical equation for longitudinal dispersion coefficient as

$$D = 0.058\frac{hU}{S} \qquad \dots (6)$$

Magazine et al. [9] experimentally studied the effect of large-scale bed and side roughness on dispersion. They derived an empirical predictive equation for the estimation of dimensional dispersion coefficient using roughness parameters of the channel, such the Reynolds number, details of boundary size, and spacing of roughness elements to account for blockage effects. Based on the experimental results of their study and an analysis of the available existing dispersion data, they developed the following expression:

$$\frac{D}{RU} = 75.86P^{-1.632} \qquad \dots. (7)$$

in which P= a generalized roughness parameter incorporating the influence of the resistance and blockage effects, which are result of the roughness elements. For the prediction of dispersion coefficient in natural streams, Magazine et al. [9] proposed the following equation:

$$P = 0.4\frac{U}{U_*} \qquad \dots.. (8)$$

Asai and Fujisaki [10] examined the dependence of the longitudinal dispersion coefficient on the width-to-depth ratio by using the k-ε model. They showed that the dispersion coefficient increases as the width-to-depth ratio increases up to 20; as the width-to-depth ratio increases further, the dispersion coefficient tends to decrease. Iwasa and Aya [11], by analyzing their laboratory data and previous field data collected by Nordin and Sabol [12] and others, derived an equation to predict the dispersion coefficient in natural streams and canals. The result is

$$\frac{D}{hU_*} = 2.0\left(\frac{W}{h}\right)^{1.5} \qquad \dots. (9)$$

Gubashi, et al.[13,14] derived from series of laboratory experiments conducted on an open channel the following equation :

$$D = 17.018 * hU_* + 0.0035 \qquad \dots(10)$$

COMPARISON WITH STREAM DATA

In order to test the behavior of the existing dispersion coefficient equations, 24 data sets measured in flume of hydraulic laboratory of the Engineering at Al-Mustansirya University were collected (see Gubashi, et al. [13,14]. These data sets contain hydraulic parameters including mean depth, mean velocity, slope and width of the flume.

To calculate the observed dispersion coefficient from field data, the moment method was considered. The field data sets with measured dispersion coefficients are listed in Table 1.

Among the methods for predicting dispersion coefficient suggested by previous investigators, five simple theoretical and empirical equations were tested using 24 field data sets. These included the dispersion equations proposed by Elder [3], McQuivey and Keffer [8] Fischer [7], Magazine et al. [9], and Gubashi et al. [13]. The dispersion coefficients that were calculated using the selected equations were compared with measured data and are shown in Figs. 1 to 5. In these figures., D_p is the predicted dispersion coefficient, and D_m is the measured dispersion coefficient.

These figures show that the use of Elder's equation significantly underestimates measured values(see Fig. 5) , whereas McQuivey an Keffer's equation (Fig. 2), Fischer's equation (Fig. 4) and Magazine equation (Fig. 3) generally overestimates. The equation of Gubashi et al. (Fig. 1) predicts values which agree relatively well with measured values.

To evaluate the difference between measured and predicted values of the dispersion coefficient more quantitatively, discrepancy ratio which is defined by White et al. [15] is used as an error measure.

$$\text{Discrepancy Ratio} = \log \frac{D_p}{D_m}$$

If the discrepancy ratio is 0, the predicted value of the dispersion coefficient is identical to the measured dispersion coefficient. If the discrepancy ratio is larger than 0, the predicted value of the dispersion coefficient overestimates, and if the discrepancy ratio is smaller than 0, it underestimates. Accuracy is defined as the proportion of numbers for which the discrepancy ratio is between -0.3 and 0.3 for the total number of data.

Discrepancy ratios for each equation for the 24 field data sets are shown in Figs. 6 to 10. These figures show that equation of Gubashi et al. (Fig. 6) is more accurate than the other equations.

DEVELOPMENT OF NEW EQUATION

Major factors which influence dispersion characteristics of pollutants in open channels can be categorized into three groups: fluid properties, hydraulic characteristics of the channel, and geometric configurations [16]. The fluid properties include fluid density, viscosity, and so on. The cross-sectional mean velocity, shear velocity, and the depth of flow can be included in the category of bulk hydraulic characteristics. The bed forms can be regarded as the geometric configuration. The dispersion coefficient can be related to these parameters as:

$$D = f_1 (\rho, \mu, U, U_*, h, S_f) \qquad …. (11)$$

in which ρ = fluid density; μ = fluid viscosity; and S_f = bed shape factor.

By using dimensional analysis, a new functional relationship between dimensionless terms was derived as

$$\frac{D}{hU_*} = f_2\left(\circ\frac{Uh}{\ll}, \frac{U}{U_*}, S_f \right) \qquad … (12)$$

in which $\frac{D}{hU_*}$ = dimensionless dispersion coefficient; $\circ\frac{Uh}{\ll}$ = Reynolds number; $\frac{U}{U_*}$ = friction term . S_f = bed shape factor. In this study, this parameter was dropped because it represent parameter not easily collected for open channel, and furthermore, the influence of this parameter can be included in the friction term. Thus Eq. (12) reduces to

$$\frac{D}{hU_*} = f_3\left(\circ\frac{Uh}{\ll}, \frac{U}{U_*} \right) \qquad ……… (13)$$

This functional relationship indicates that dispersion coefficient is dependent only on hydraulic parameters. These parameters are depth of flow, h, mean velocity, U, shear velocity, U_*, and fluid properties.

To test the correlation between the dimensionless dispersion coefficient and dimensionless parameter included in Eq. (13), plots of measured dispersion coefficient versus measured hydraulic parameters were constructed using arithematic scale. The plot of dimensionless dispersion coefficient versus Reynolds number is shown in Fig. (11). This Fig. shows that, for the data collected in open channel, the Reynolds number has an insignificant effect for fully turbulent flow on the dimensionless dispersion coefficient. This confirms the assumption that, for turbulent flow in rough open channel, the effect of Reynolds number is probably negligible.

The plot of D/hU* versus U/U* is shown in Fig. (12).This figure demonstrates that the dimensionless dispersion coefficient appears to have some dependency on the friction term.

REGRESSION METHOD

A standard nonlinear multiple model is prepared by the writer in which dependent variable Y is related to N unknown independent variables X which can be given as:

$$Y = aX_1^b X_2^c X_3^d ....X_N^z \quad ...... (14)$$

in which X=independent variables which represent the hydraulic parameters; a, b, c, …z =unknown regression coefficients. Taking logarithms of Eq. (14), a linear multiple form can be derived as follows:

$$\ln Y = \ln a + b \ln X_1 + c \ln X_2 + d \ln X_3 \; ....+z \ln X_N \quad .... (15)$$

The solution of Eq. (15) is usually obtained by a least-squares method in which a sum of the squares of the residuals is minimized. Eq. (15) is transformed as:

$$Y_o = A + B L_1 + C L_2 + D L_3 \; ....+ZL_N \quad ...(16)$$

By using the least-square error method, the normal standard equations are resulted as:

$$\sum Y_o = nA + B\sum L_1 + C\sum L_2 + D\sum L_3 \quad .... \quad .... (17)$$

$$\sum L_1 Y_o = A\sum L_1 + B\sum L_1^2 + C\sum L_1 L_2 + D\sum L_1 L_3 .... \quad .... (18)$$

$$\sum L_2 Y_o = A\sum L_2 + B\sum L_1 L_2 + C\sum L_2^2 + D\sum L_2 L_3 \text{..............} \qquad \text{............ (19)}$$

$$\sum L_3 Y_o = A\sum L_3 + B\sum L_1 L_3 + C\sum L_2 L_3 + D\sum L_3^2 \qquad \text{..............(20)}$$

$$\sum L_N Y_o = A\sum L_N + B\sum L_1 L_N + C\sum L_N L_2 + D\sum L_N L_3 \text{.........} + Z\sum L_N^2 \qquad \text{..................... (21)}$$

A basic program is prepared to solve this model and calculation parameters of multiple regression A, B, C, ...Z. In this study, the solution of the above linear equations is made using Gaussian elimination method. The flow chart explaining this procedure is shown in Fig.(13).

NEW DISPERSION EQUATION

In this study, a nonlinear multi-regression equation for predicting the dimensionless dispersion coefficient as a function of the friction term and Reynolds number is derived by using nonlinear multiple model . The data sets used in the development of the new dispersion coefficient equation are the same as those used in the comparison of the previous dispersion coefficient equations. Among 24 data sets, 12 measured data sets (see Table 1) were selected to derive the dispersion coefficient, and 12 measured data sets were used to verify the new dispersion coefficient equation.

The new regression equation derived by using a nonlinear multiple regression model is given as:

$$\frac{D}{hU_*} = 14.723\left(\frac{U}{U_*}\right)^{-7.224} \mathrm{Re}^{1.684} \qquad \text{..... (22)}$$

In deriving Eq. (22), the correlation coefficient is 0.87.

VERIFICATION

Twelve measured data sets that were not used in the derivation of the regression equation are used to verify the proposed equation (22) for predicting dispersion coefficient. The dispersion coefficients predicted by the proposed equation and the existing equations are compared with measured dispersion coefficients. One existing dispersion equation that was proven to be relatively better than other equations in predicting dispersion coefficient in open channel is selected; this is Gubashi et al. [13] equation .

The comparisons of estimated dispersion equations with measured data are shown in Fig. 14. This figure shows that the proposed equation (22) predicts quite well, whereas Gubashi et al. [13] equation underestimate in some cases.

A discrepancy ratio of new equation for 24 field data sets is shown in Fig. 15. The proposed equation predicts better than the equation of Gubashi et al. [13], and the discrepancy ratio of the new dispersion coefficient equation ranges from -0.16 to 0.3. These results demonstrate that the new dispersion coefficient equation developed in this study is superior to the existing equations in predicting dispersion coefficient more precisely in open channel.

CONCLUSIONS

The results of this study show that, among the existing dispersion coefficient equations, Elder's equation is not amenable to estimate the dispersion coefficient of the 1D dispersion model because it underestimates significantly. Gubashi et al. [13] equation predict good estimate, whereas the equations of McQivey and keefer [8], Fischer [7], and Magazine et al. [9] overestimate significantly.

In addition to the comparative analysis of previous theoretical and empirical equations, a new, simple method for predicting dispersion coefficients by using hydraulic parameters, which are easily obtained for open channel, has been developed. Dimensional analysis was implemented to select physically meaningful parameters that are required for the new equation in order to predict longitudinal dispersion in open channel. The nonlinear multiple model has been prepared to derive a new dispersion coefficient equation. The proposed equation allows superior prediction as compared to the existing equations, and the discrepancy ratio of the new dispersion coefficient equation ranges from -0.16 to 0.3. The dispersion coefficient estimated by the proposed equation can be used when the 1D dispersion model is applied to open channel where mixing and dispersion data has not been collected, and thus the measured dispersion coefficient is not available.

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Table (1) Results of longitudinal dispersion coefficients.

EXP. NO.	Flow Discharge (L/s)	Average Velocity (m/s)	Second Moment Mehtod $D(m^2/s)$	Shear Velocity* Depth (m^2/s)
1	0.9	0.05923	0.029824	0.000283
2	1	0.062585	0.02109	0.000307
3	1.1	0.06482	0.042657	0.000331
4	1.24	0.06925	0.034211	0.000357
5	3.05	0.11971	0.008361	0.000609
6	3.06	0.120235	0.013611	0.000609
7	4.43	0.14864	0.011592	0.000767
8	4.58	0.149445	0.021554	0.0008
9	5.1	0.15764	0.012282	0.000868
10	5.76	0.169485	0.015896	0.000938
11	5.98	0.17163	0.02261	0.000973
12	6.2	0.1738	0.02655	0.001009
13	6.26	0.175355	0.027537	0.001009
14	6.85	0.1831	0.029083	0.001082
15	7.71	0.19296	0.028074	0.001194
16	8.02	0.19653	0.027133	0.001232
17	8.7	0.204555	0.02954	0.00131
18	9.08	0.209385	0.02356	0.00135
19	9.34	0.211335	0.026454	0.00139
20	9.65	0.21409	0.026021	0.00143
21	10.05	0.21903	0.033231	0.001471
22	11.78	0.23491	0.0307	0.001679
23	12.22	0.23965	0.02548	0.001722
24	12.24	0.240045	0.02905	0.001722

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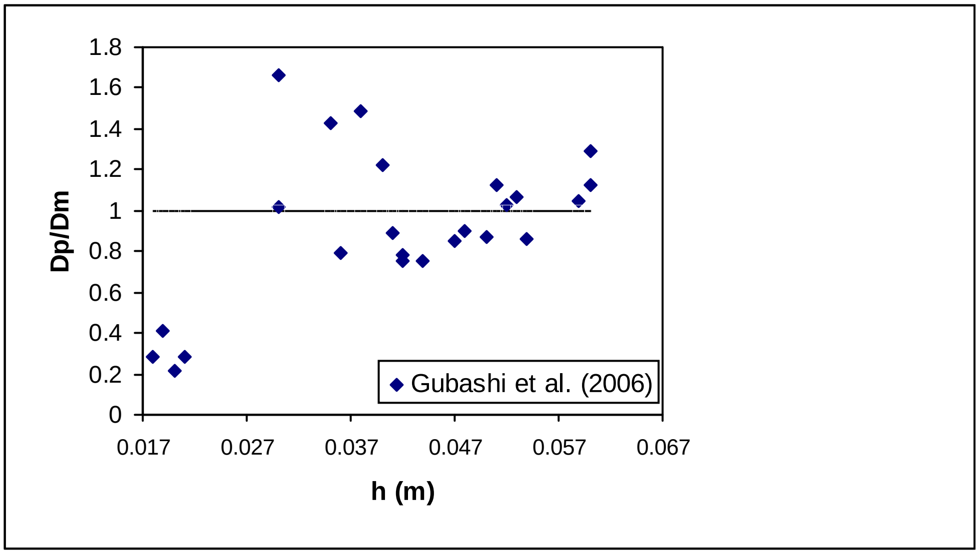

Fig. 1 Comparison of Estimated Dispersion Coefficients by Gubashi et al (2006) With Measured Data.

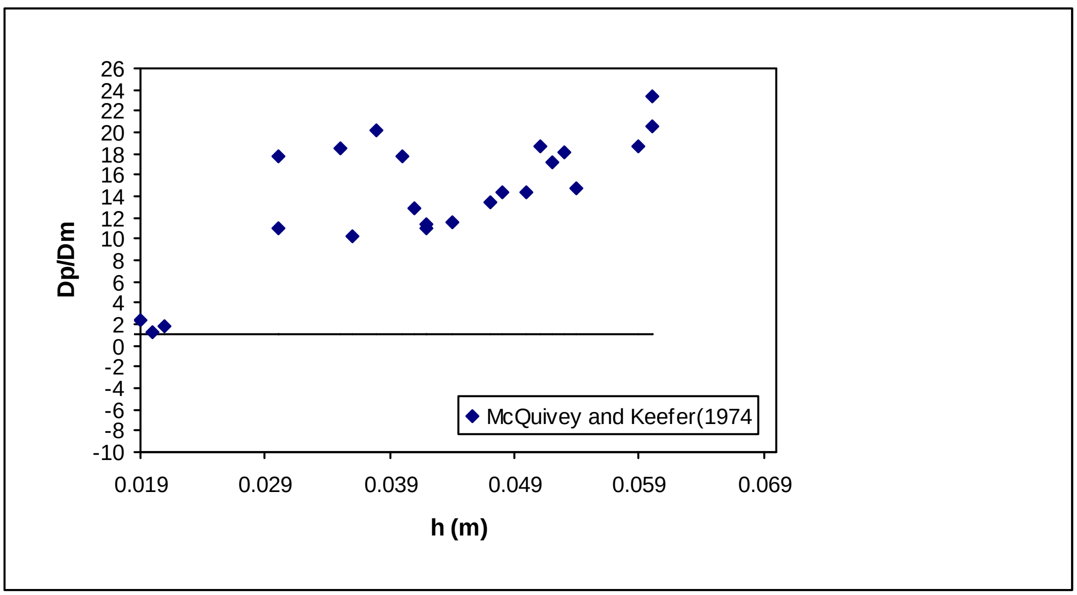

Fig. 2 Comparison of Estimated Dispersion Coefficients by McQuivey and Keefer (1974) With Measured Data.

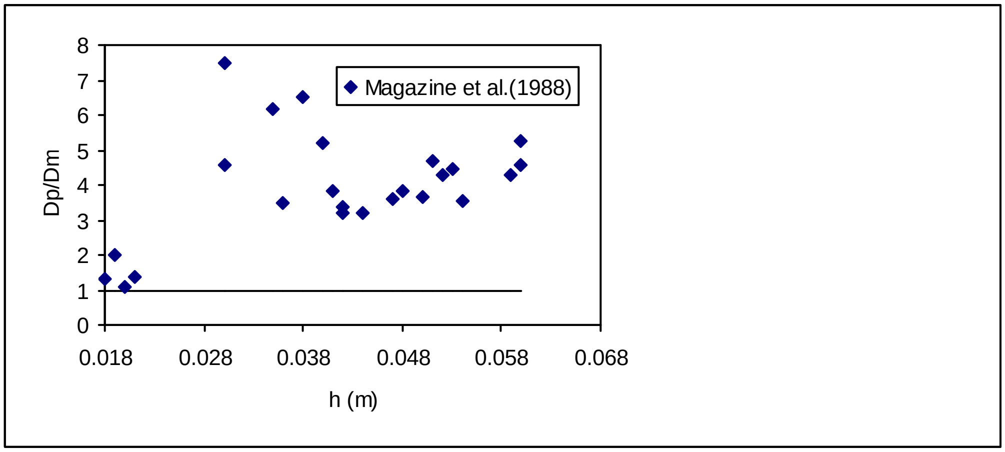

Fig. 3 Comparison of Estimated Dispersion Coefficients by Magazine et al. (1988) With Measured Data.

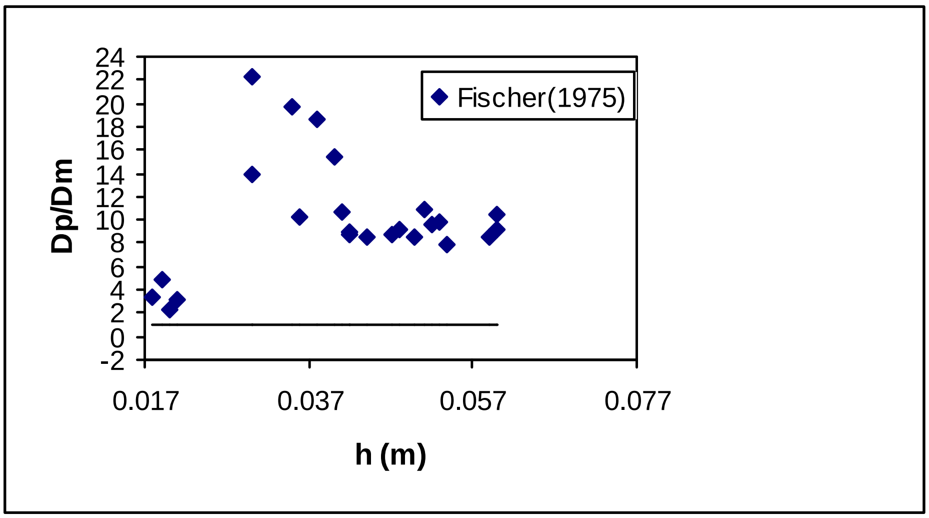

Fig. 4 Comparison of Estimated Dispersion Coefficients by Fischer (1975) With Measured Data.

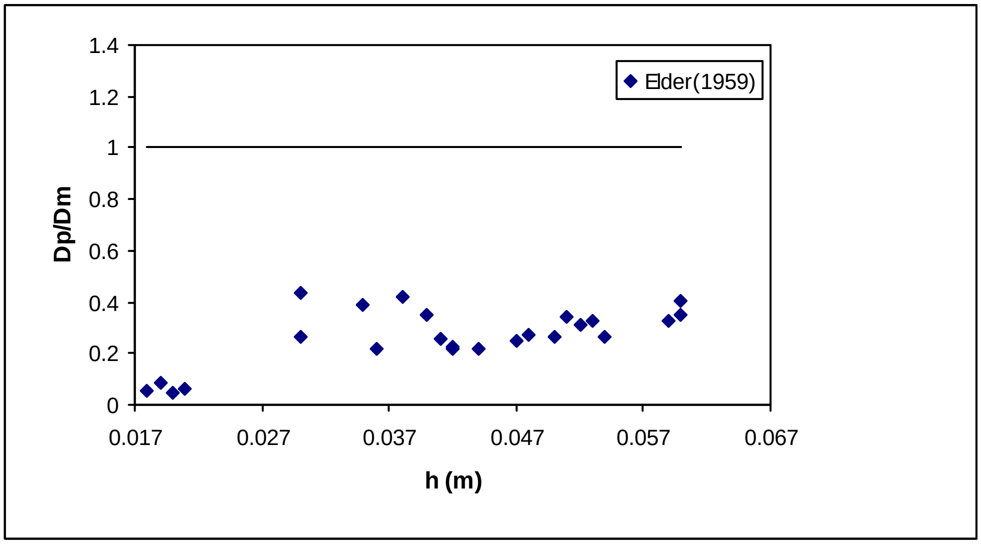

Fig. 5 Comparison of Estimated Dispersion Coefficients by Elder (1959) With Measured Data

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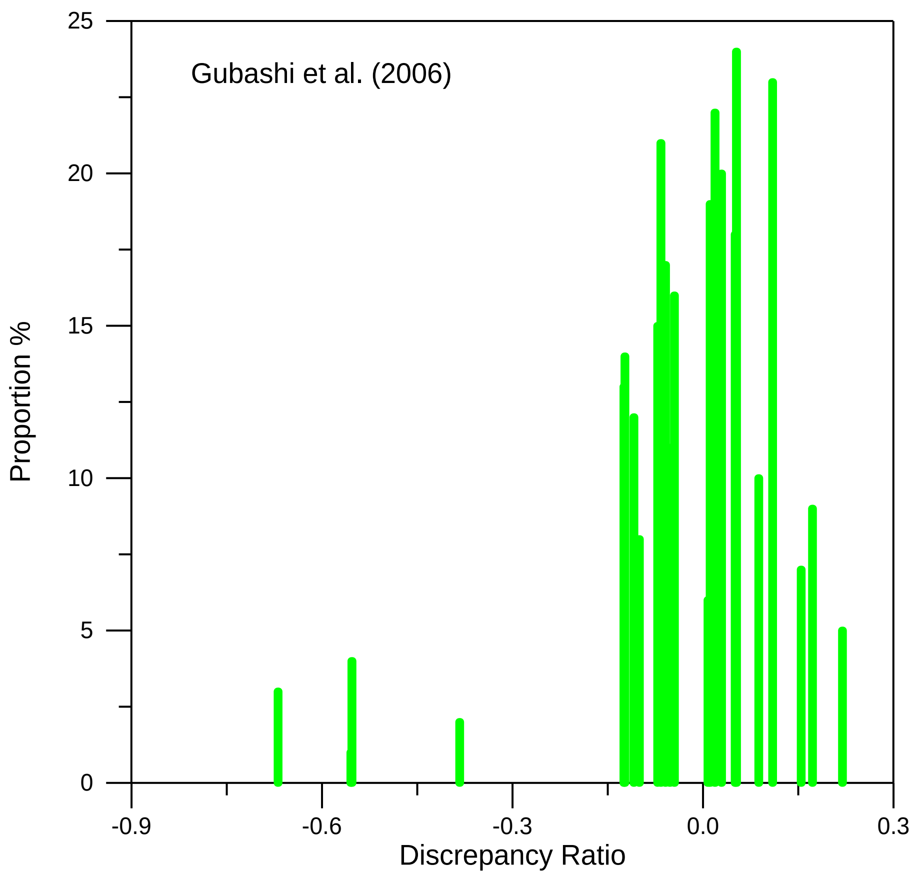

Fig 6 Comparison of Discrepancy Ratios of Eq. Proposed by Gubashi et al.(2006)

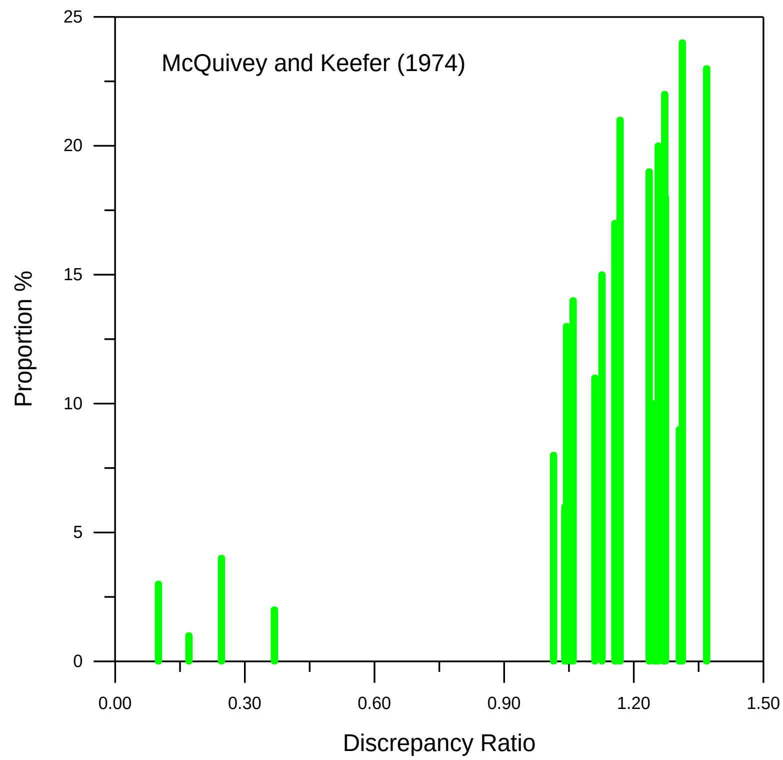

Fig 7 Comparison of Discrepancy Ratios of Eq. Proposed by McQuivey and Keefer (1974)

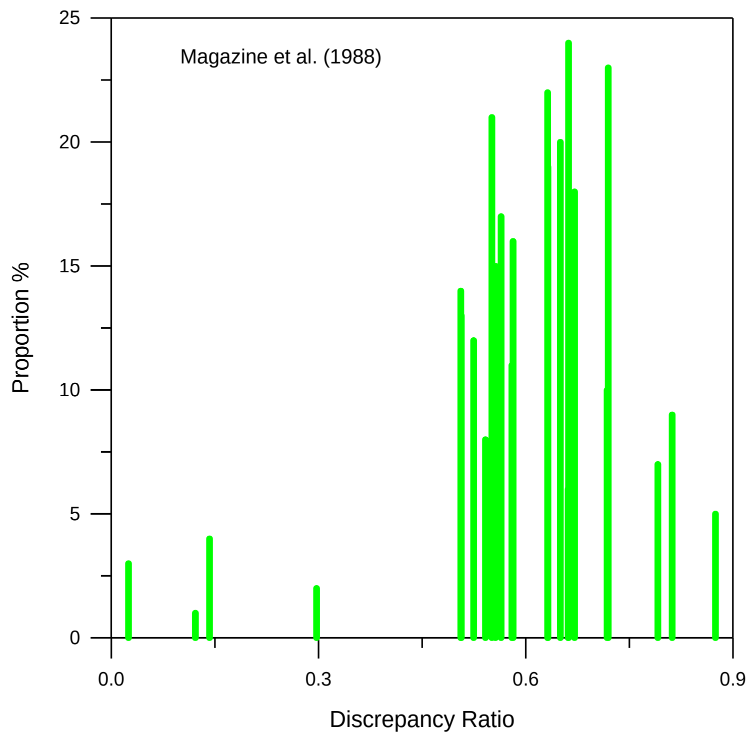

Fig 8 Comparison of Discrepancy Ratios of Eq. Proposed by Magazine et al. (1988)

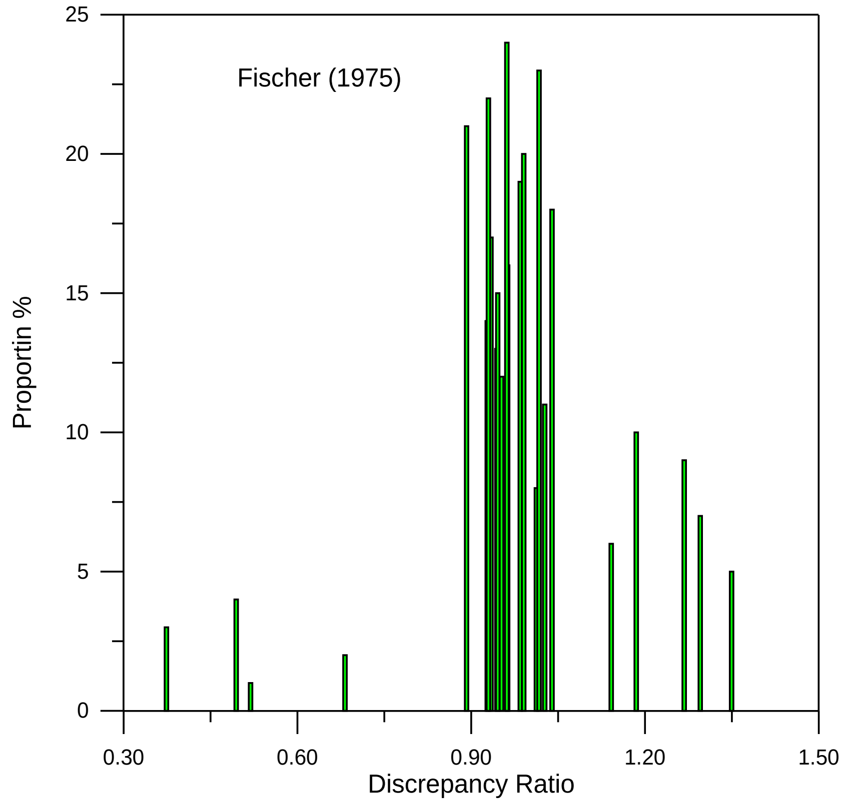

Fig 9 Comparison of Discrepancy Ratios of Eq. Proposed by Fischer (1975)

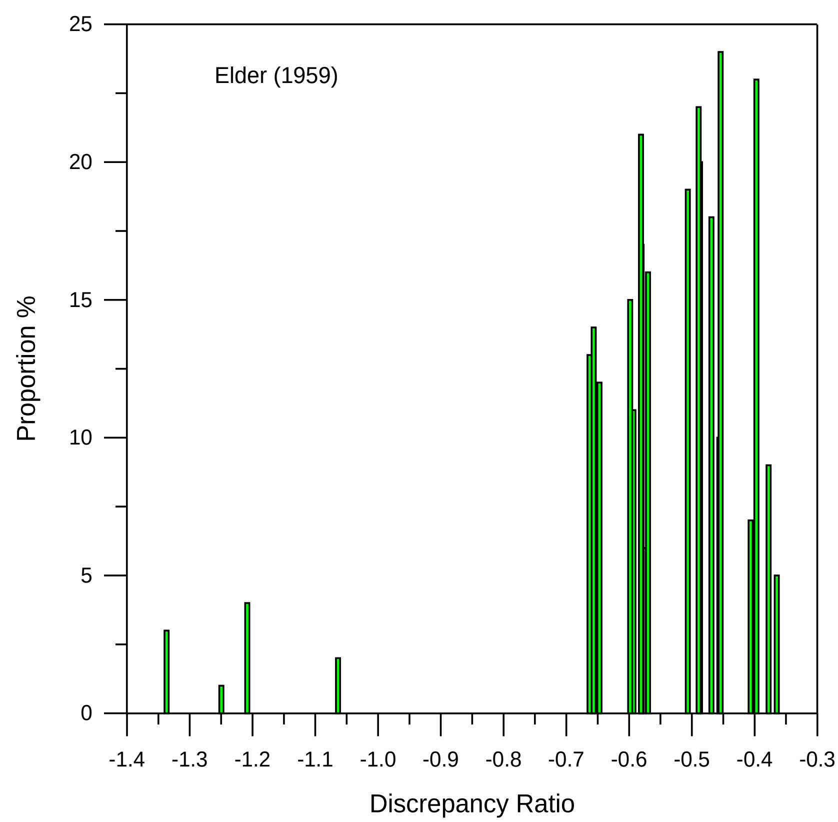

Fig 10 Comparison of Discrepancy Ratios of Eq. Proposed by Elder (1959).

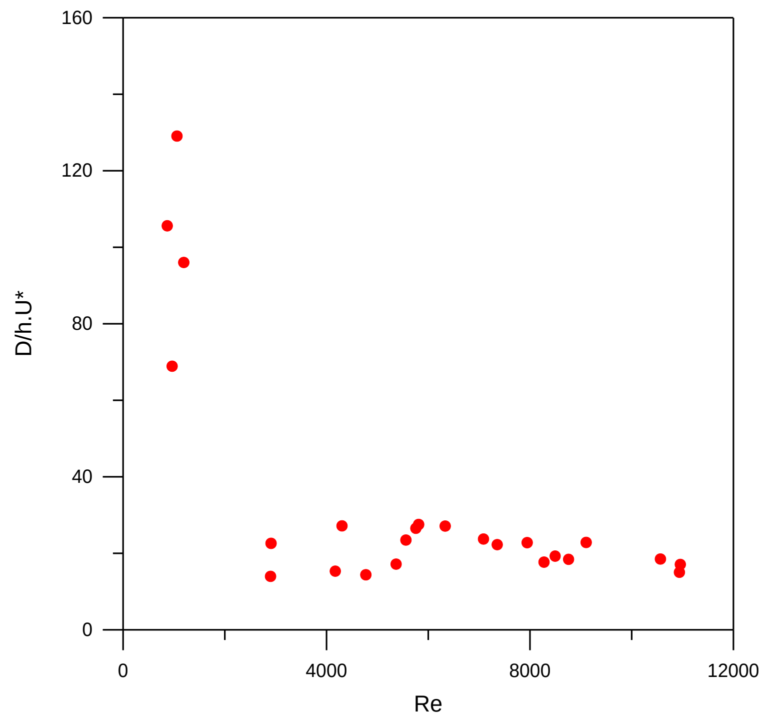

Fig. 11 Plots of Dimensionless Dispersion Coefficient (D/hU*) Versus Reynolds Number.

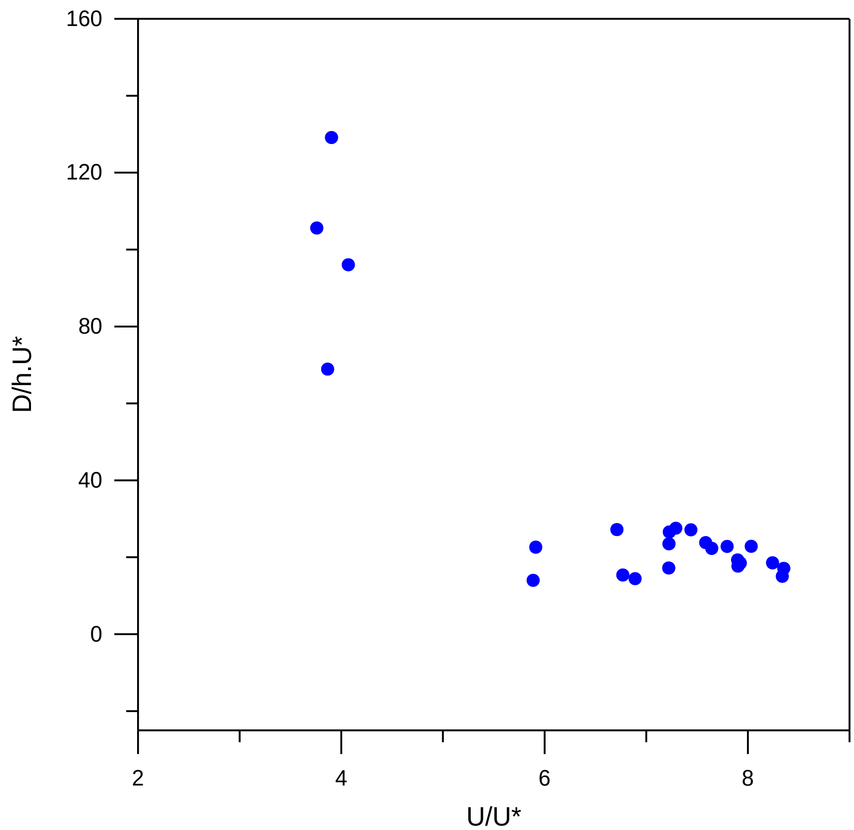

Fig. 12 Plots of Dimensionless Dispersion Coefficient (D/hU*) Versus Friction Term (U/U*).

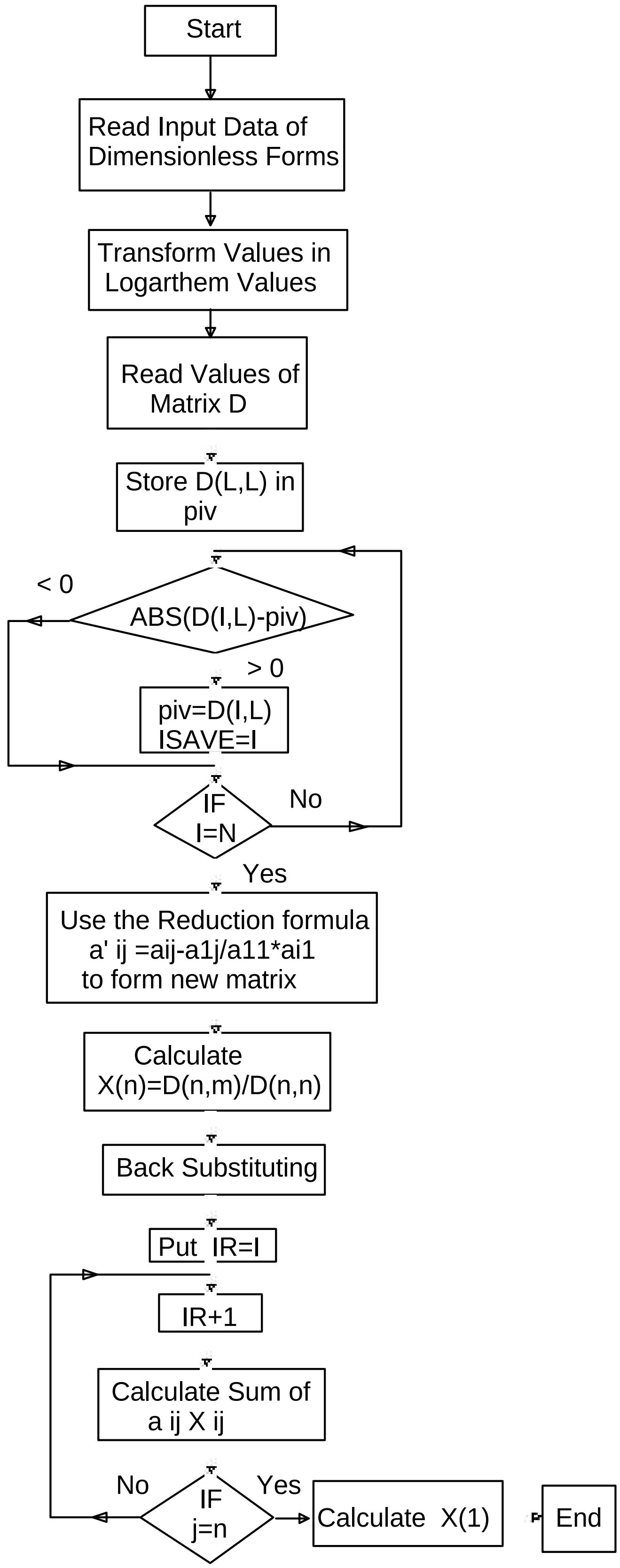

Fig. 13 Flow chart of Nonlinear Multiple Regression Analysis.

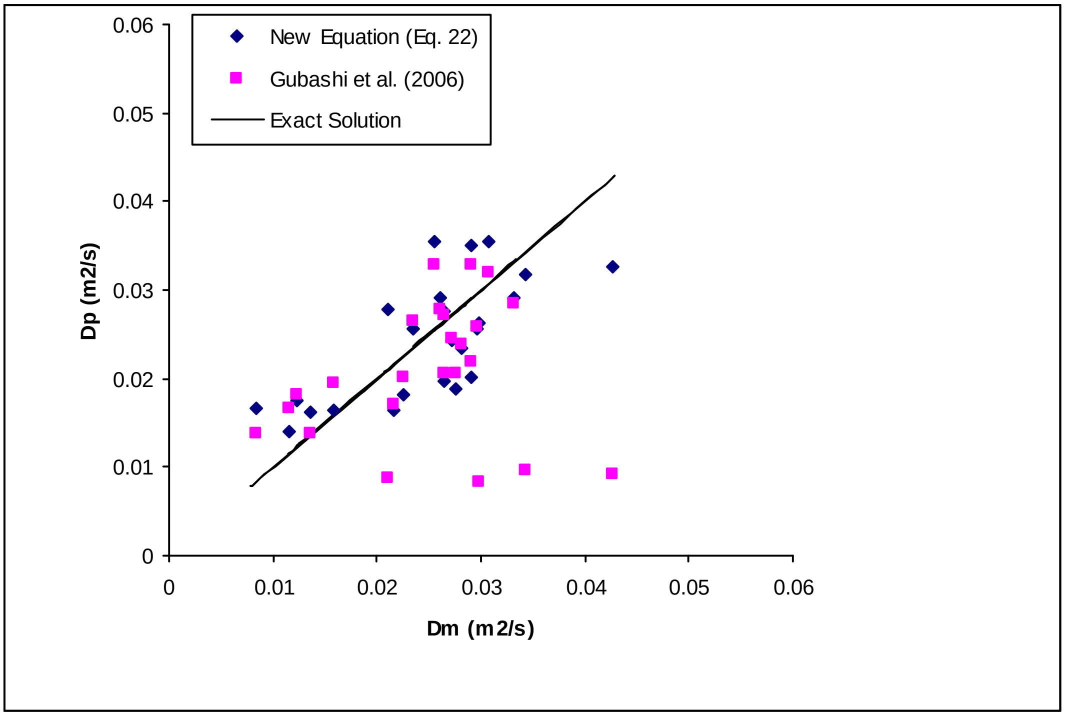

Fig. 14 Comparison of Estimated Dispersion Coefficients With Measured Data Used in Verification

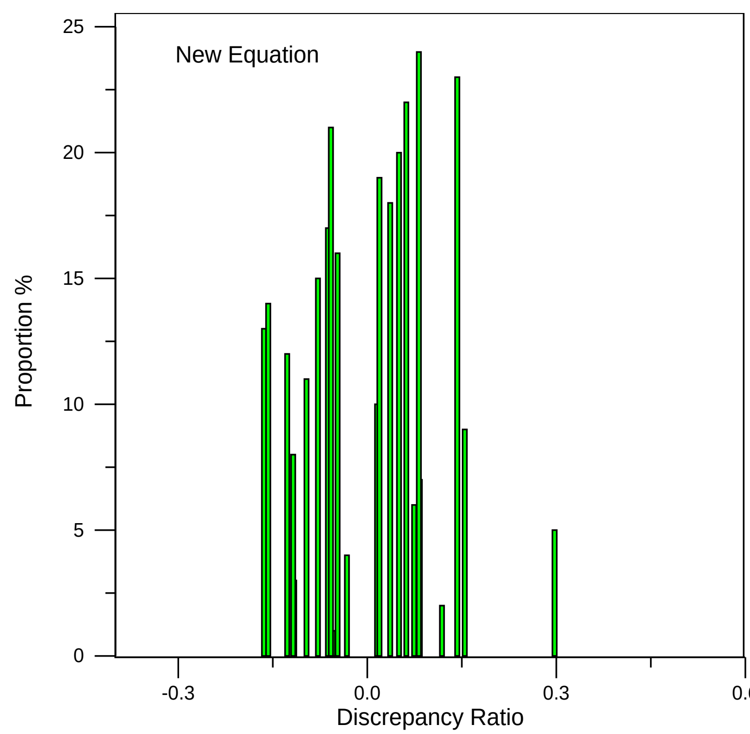

Fig. 15 Comparison of Discrepancy Ratios of New Equation.

استخدام المتغيرات الهيدروليكية لتقدير معامل التشتت الطولي في القناة المفتوحة

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الخلاصة

أجريت مقارنة تحليلية بين معادلات تجريبية ونظرية سابقة لبيان دقتها في التنبؤ لمعامل التشتت في القنوات المفتوحة. تم تطوير معادلة جديدة للتنبؤ لمعامل التشتت باستخدام المتغيرات الهيدروليكية وأعتمدت من أجل ذلك طريقة الانحدار المتعدد الغير خطي لاشتقاق تلك المعادلة وبعد أجراء المقارنة تبين ان هذه المعادلة ذات دقة عالية في التنبؤ لمعامل التشتت في القناة المفتوحة عند مقارنتها مع مثيلاتها من المعادلات السابقة.

الكلمات الدالة: التشتت الطولي , المتغيرات الهيدروليكية , معاملات التشتت