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Employment of Regression-Based Decision Tools to Predict the Shear Capacity of Reinforced Concrete Beams Without Web Reinforcement

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Keywords:
Shear capacity prediction; Reinforced concrete beams; Regression models; Stepwise multivariate regression; Web reinforcement-free beams.

Highlights:

- *Predicts shear capacity of RC beams without web reinforcement using regression models.*
- *Stepwise multivariate regression (SMR5) achieves lowest RMSE (17.8 MPa).*
- *Full linear regression (LR-F) shows highest accuracy (R^2 = 0.9733).*
- *Longitudinal reinforcement area (A_{sl}) has the strongest influence on shear strength.*

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Abstract: Shear failure in the reinforced concrete (RC) beams with no web reinforcement is a structural problem due to its sudden nature and absence of precursors. The purpose of this research is to predict and assess the shear capacity (Vc) of the beams by applying several statistical regression models. Different combinations of a full linear regression model, a simplified linear model, and stepwise multivariate regression (SMR) models were formulated, tested, and compared. For training and validation, a dataset containing 398 RC beams with different geometries, material properties, and loading configurations was obtained. Other key factors included the beam width, effective depth, reinforcement area, concrete compressive strength, and the shear span-to-depth ratio. The model was developed in Python and SPSS, and the outcomes were evaluated based on R^2, RMSE, MAE, classification accuracy, and residual analysis. The results indicated that the full linear regression model retained the best predictive performance as indicated by an R^2 score of 0.9733. However, the fifth-order SMR model scored the lowest RMSE of 17.8 MPa. Furthermore, the simplified linear model greatly underestimated the strength and performed poorly in its predictive functionality. The study emphasizes that stepwise regression model building improves the accuracy of the model while maintaining clear practical relevance. This research enables engineers to make decisions based on reliable data.

استخدام أدوات القرار القائمة على الانحدار للتنبؤ بقيمة مقاومة القص للكمرات الخرسانية المسلحة بدون تسليح قص

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الخلاصة

فشل القص في الكمرات الخرسانية المسلحة (RC) التي تخلو من تسليح القص يمثل مشكلة إنشائية بسبب طبيعته المفاجئة وغياب المؤشرات المسبقة. يهدف هذا البحث إلى التنبؤ بتقييم قدرة مقاومة القص (Vc) للكمرات باستخدام عدة نماذج إحصائية للانحدار. تم صياغة واختبار ومقارنة نماذج مختلفة تشمل نموذج الانحدار الخطي الكامل، ونموذج الانحدار الخطي المبسط، ونماذج الانحدار المتعدد التدريجي. (SMR) لتدريب واختبار النماذج، تم استخدام مجموعة بيانات تحتوي على ٣٩٨ كمرة خرسانية مسلحة ذات أبعاد هندسية وخصائص مواد وتكوينات تحميل مختلفة. تضمنت العوامل الرئيسية عرض الكمرة، والعمق الفعال، ومساحة التسليح، ومقاومة الضغط للخرسانة، ونسبة الامتداد القص إلى العمق. تم تطوير النموذج باستخدام لغة Python وبرنامجSPSS ، وتم تقييم النتائج بناءً على معامل التحديد(R^2) ، وجذر متوسط مربعات الخطأ(RMSE) ، ومتوسط الخطأ المطلق(MAE) ، ودقة التصنيف، وتحليل البواقي. أظهرت النتائج أن نموذج الانحدار الخطي الكامل حقق أفضل أداء تنبؤي بمعدل R^2 بلغ ٠,٩٧٣٣ . ومع ذلك، سجل نموذج الانحدار المتعدد التدريجي من الدرجة الخامسة (SMR) أقل قيمة لـ RMSE عند ١٧,٨ ميجا باسكال. من ناحية أخرى، قلل نموذج الانحدار الخطي المبسط من تقدير القوة بشكل كبير وأظهر أداءً ضعيفًا في الوظيفة التنبؤية. تؤكد الدراسة أن بناء نموذج الانحدار التدريجي يحسن دقة النموذج مع الحفاظ على الأهمية العملية الواضحة. يمكّن هذا البحث المهندسين من اتخاذ قرارات تستند إلى بيانات موثوقة.

الكلمات الدالة: تنبؤ مقاومة القص، الكمرات الخرسانية المسلحة، نماذج الانحدار، الانحدار المتعدد التدريجي، الكمرات غير المسلحة قصياً.

1.INTRODUCTION

The mechanical behavior of structural elements within reinforced concrete (RC) structures determines the safety, functional operation, and stability of beams. The standard practice for designing RC beams includes using longitudinal and transverse reinforcement to withstand flexural and shear forces in most practical applications. Real-world construction of beams often occurs without web (transverse) reinforcement due to design limitations or cost constraints and architectural requirements. A design choice that omits web reinforcement makes structures more susceptible to brittle shear failure, as this mode of failure produces minimal deformation and no warning signs. The structural engineering community continues to focus on accurate shear strength predictions for beams because this topic remains essential. The shear behavior of RC beams consists of multiple interacting mechanisms, including diagonal tension cracking, aggregate interlock, dowel action of longitudinal reinforcement, and arching action. The absence of web reinforcement makes concrete, along with longitudinal bars, more crucial to the structure's behavior. The beam strength depends heavily on beam width (b_w) and effective depth (d), along with compressive strength of concrete (f'c), reinforcement ratio (ρ), and shear span-to-depth ratio (a/d). Shear capacity predictions for such conditions prove more complex than flexural strength predictions because of the interacting nonlinear mechanisms and coupled effects at play. Current building code shear design provisions, including ACI 318-25 [1], Eurocode 2 [2], and CSA A23.3 [3], employ empirical or semi-empirical mathematical approaches. The application of these equations remains straightforward, yet their effectiveness declines when used beyond their calibrated range, particularly for beams that lack web reinforcement. The result of this approach leads to either extremely conservative design choices or dangerous structural evaluations when working with advanced or unconventional applications. The combination of extensive experimental data with modern statistical learning tools provides a promising solution to traditional code-based predictions. Machine learning and regression-based models identify intricate patterns along with nonlinear connections in experimental data, which standard analytical methods struggle to detect. Multi-scale regression models, which span linear regression (LR) to nonlinear regression (NLR) and classification models, including multi-logistic regression (MLR), enable accurate and generalizable predictions for RC beams. These models provide a flexible framework for considering material properties in conjunction with geometric parameters and loading conditions, leading to more precise predictions of structural behavior. Despite the growth in the use of such models for predicting compressive or flexural strength, limited attention has been given to modeling the shear capacity of beams without web reinforcement using this systematic, comparative modeling approach. Previous works have focused on specific types of concrete, e.g., ultra-high-performance fiber-reinforced concrete, specialized reinforcement schemes, or machine learning black-box models with limited interpretability. Still, there is a need for clear, statistically sound, and interpretable models that can provide both accurate numerical predictions and practical decision-making support for engineers. The present study aims to employ statistical multi-scale regression models to predict the shear capacity of RC beams without web reinforcement. By leveraging a comprehensive dataset collected from previously published experimental studies, the proposed models incorporate key influencing parameters, such as beam dimensions, concrete compressive strength, reinforcement ratio, and shear span-to-depth ratio. The performance of the multi-scale models is evaluated using standard statistical metrics and compared against traditional design code predictions to highlight their advantages in terms of accuracy and generalizability. This research aimed to contribute to the development of robust predictive tools that can assist engineers in designing safer and more economical RC structures, while also paving the way for the integration of data-driven intelligence into structural engineering practice. The motivation behind this research stems from the code-based equations' lack of consistency and accuracy when applied to beams without stirrups, as well as the fact that experimental testing is expensive, time-consuming, and not always feasible during design or assessment. In addition, statistical regression techniques provide a transparent, flexible, and efficient alternative for developing predictive equations. Also, model comparisons are rarely made within a single study, leaving a gap in identifying the most suitable modeling strategy for this type of structural problem. Shear capacity prediction in reinforced concrete (RC) beams without web reinforcement has been a significant focus because shear failure in these structures occurs suddenly and in a brittle manner. The current design codes fail to accurately represent the intricate relationships between variables that control shear behavior in beams without transverse reinforcement. The Modified Compression Field Theory (MCFT) and its simplified version (SMCFT) represent advanced analytical approaches for studying the behavior of concrete and steel under combined stress conditions (Vecchio & Collins [4], Bentz et al. [5]). The models provide strong theoretical foundations but require iterative computational methods and specific assumptions to function, which may not hold universally across all beam geometries or loading scenarios. To overcome the shortcomings of both code-based models and analytical models, data-driven methods with artificial intelligence (AI) and machine learning (ML) models have been proposed by researchers. Artificial neural networks (ANNs), support vector machines (SVMs), gene expression programming (GEP), and decision trees have demonstrated significant capabilities for modeling nonlinear input relationships (Mansour et al. [6]; Khademi et al. [7]; Chou et al. [8]). While such models frequently achieve higher predictive accuracy compared to traditional approaches, they are typically trained for a single scale of data representation. They may fail to provide interpretations that are useful in engineering applications. Meanwhile, hierarchical regression models are particularly effective when data are correlated, or clustered, such as road beams within bridge frames or samples across testing laboratories. They can handle both fixed (e.g., material properties) and random (e.g., construction variation) effects, leading to a more detailed view of the behaviors of structures (Raudenbush [9]; Motlagh and Naghizadehrokni [10]). Applications range from extrapolating compressive strength across different projects to calculating load-bearing capacity with site-specific modifiers. Their approach involves the use of artificial neural networks, random forest regression, and polynomial regression methods to enhance prediction accuracy, particularly in the context of hierarchical and multi-resolution regression models in HSSP. Multi-scale regression methodologies, leveraging wavelet transforms or decomposition frameworks, allow local and global structural behaviors to be addressed at the same time, thereby enhancing the understanding of stress distribution, crack initiation and detection, and damage propagation (Hou et al. [11]; Arbaoui et al. [12]). Recent research has shifted towards hybrid approaches that interfuse hierarchy and multi-resolution models, typically via a Bayesian treatment. These models combine data at the spatial or functional level and can measure uncertainty in performance predictions, which is of great importance for structural health monitoring or fragility assessment. Bayesian hierarchical regression, including Gaussian process-based methods, is gaining traction for integrating data from various sources, e.g., simulations, sensors, and inspections, while capturing probabilistic variation in material and structural behavior. One such study is by Huang et al. [13], who proposed a Bayesian system identification method based on hierarchical sparse Bayesian learning and Gibbs sampling for structural damage assessment. Their methodology effectively detects, locates, and quantifies structural damage using incomplete modal data, addressing challenges associated with uncertainty and sparse measurements. Nevertheless, a shortage of research remains focused specifically on statistical multi-scale regression models for RC beams without stirrups. Most existing studies either overlook scale interactions or fail to quantify uncertainty in input data. Moreover, comparative analyses between such models and established design codes are often lacking. This paper addresses this gap by developing and evaluating three structured models, Linear Regression (LR), Nonlinear Regression (NLR), and Multi-Logistic Regression (MLR), to predict the Shear capacity (Vc) of RC beams without web reinforcement. Unlike prior machine learning approaches that prioritize accuracy at the expense of interpretability, the present study focuses on multi-scale regression models that are both accurate and transparent, offering clear insights into the structural behavior of RC beams without stirrups. In addition, prior studies often relied on a single modeling technique or limited datasets, whereas this work leverages a verified and curated database of 398 experimental tests, covering a broad range of geometrical configurations, material strengths, and loading conditions. Therefore, the proposed models are not only reliable but also highly applicable to a wide range of practical scenarios.

2.METHODOLOGY

The present study adopts a multi-model statistical approach to predict the Shear capacity (Vc) of reinforced concrete (RC) beams without web reinforcement. The methodology consists of five main stages: (i) acquisition and preparation of the experimental dataset, (ii) selection of input variables, (iii) statistical analysis, (iv) development of prediction models using regression techniques, and (v) evaluation of model performance using standardized metrics.

2.1.Experimental Dataset

The core of this research is built upon a comprehensive and curated database comprising 398 experimentally tested RC beam specimens without web reinforcement. The dataset, compiled and refined by Dr. Robert J. Frosch and colleagues, aggregates test results from over 30 independent published studies. The database includes a wide range of beam sizes, concrete strengths, reinforcement configurations, and loading types. Each record contains geometric parameters, material properties, and measured shear capacity values. Table 1 (Appendix A) presents a summary of the data collected from the mentioned experimental tests.

To maintain consistency and avoid scale effects, all measurements were converted into metric units where necessary. The final dataset included the following parameters:

- Beam width (b_w, mm)
- Effective depth (d, mm)
- Shear span (a, mm)
- Shear span-to-depth ratio (a/d)
- Longitudinal reinforcement ratio (ρ)
- Concrete compressive strength (f'c, MPa)
- Measured shear capacity (Vc, MPa)

2.2.Data Cleaning and Normalization

Missing, duplicated, or outlier records were carefully inspected and addressed. Only complete records with verified test procedures were retained. No data augmentation or synthetic records were introduced. Additionally, the values of variables were normalized using z-score scaling to aid in training and prevent bias in the regression processes.

2.3.Variable Selection

The selection of input variables was based on both structural engineering principles and statistical relevance. Six variables were chosen to represent key geometric and material properties influencing the shear capacity of RC beams without web reinforcement as follows:

- Beam width (b_w): Influences cross-sectional area and shear path.
- Effective depth (d): Affects lever arm and internal force distribution.
- Shear span (a): Impacts the magnitude of shear forces.
- Shear span-to-depth ratio (a/d): Relates to failure mode transition and cracking pattern.
- Longitudinal reinforcement ratio (ρ): Determines flexural capacity and anchorage effects.
- Concrete compressive strength (f'c): Directly correlates with the intrinsic shear resistance of concrete.

These variables were consistently used in all developed models, enabling a comprehensive and interpretable approach to modeling shear behavior. This selection also supports comparison with existing code-based and empirical formulations.

2.4.Model Formulations

Three statistical models were developed:

1- Full Linear Regression (LR-F): Assumes a direct linear relationship between all input variables and Vc to get the most accurate equation.
2- Simplified Linear Regression (LR-S): Assumes a direct linear relationship between some of the input variables and Vc to get a simple code-style equation with acceptable accuracy.
3- Stepwise Multivariate Regression (SMR): Statistically optimal regression model by iteratively adding and/or removing predictor variables based on their significance and contribution to the model's performance.

2.5.Tools and Implementation

All models were developed using the Python programming language along with packages such as `pandas`, `scikit-learn`, and `matplotlib`, for data handling, regression modeling, and visualization. Additionally, SPSS software was used to calculate the residuals. The model evaluation included both numerical and graphical outputs to assess performance.

2.6.Rationale for Model Selection

Statistical regression models were considered, as opposed to machine learning models such as neural networks or support vector machines, due to the interest in transparency, interpretability, and engineering applicability. In previous works, advanced material informatics methods have demonstrated high prediction accuracy; however, they act as a black box model, where it is not transparent in extracting physically meaningful insights or usable equations for design.

3. STATISTICAL EVALUATION

This section presents the descriptive and inferential statistical analysis conducted on the experimental dataset to evaluate the relationships between shear capacity (Vc) and the selected geometric parameters: beam width (b_w) and effective depth (d). Understanding the statistical behavior of the dataset is essential for model development, validation, and generalization.

3.1. Descriptive Statistics

Table 1 summarizes the statistical distribution of the primary variables under investigation. These include mean, standard deviation, minimum and maximum values, as well as skewness and kurtosis, which provide insights into the symmetry and peakedness of the data distributions. The beam dimensions and shear capacity values span wide ranges, confirming the representativeness of the dataset. High positive skewness and kurtosis in Vc suggest a concentration of lower-strength beams and the presence of a few outliers with very high shear capacity.

Table 1 Summary of the Statistical Distribution of the Primary Variables.

Variable	b_w (mm)	d (mm)	a/d (m/m)	dia (mm)	f_y (MPa)	As (mm²)	Rho (%)	f'c (MPa)	Vu (kN)
Min	76	110	2.41	6.35	276	103	0.14	12	14
Max	1000	2000	8.03	38	1779	7390	6.64	105	402
Mean	212	341	3.59	18.35	463	1357	2.27	40	91
Std Dev	149	243	0.92	7.23	185	1128	1.17	22	69
Skewness	4	3	1.65	0.26	5	2.43	0.68	1.20	2.05
Kurtosis	16	12	3.62	-0.87	30	7.34	0.61	0.23	4.68

3.2. Visual Analysis

Histogram plots of all variables show non-normal distributions, especially for Vu, d, b_w, and f'c, which is heavily right-skewed, as shown in Fig. 1. However, the intervals with the highest frequency are the ones most commonly used in the construction industry, such as the concrete compressive strength between 23 and 34 MPa, and beam width between 147 and 218 mm, which represents a realistic statistical sampling.

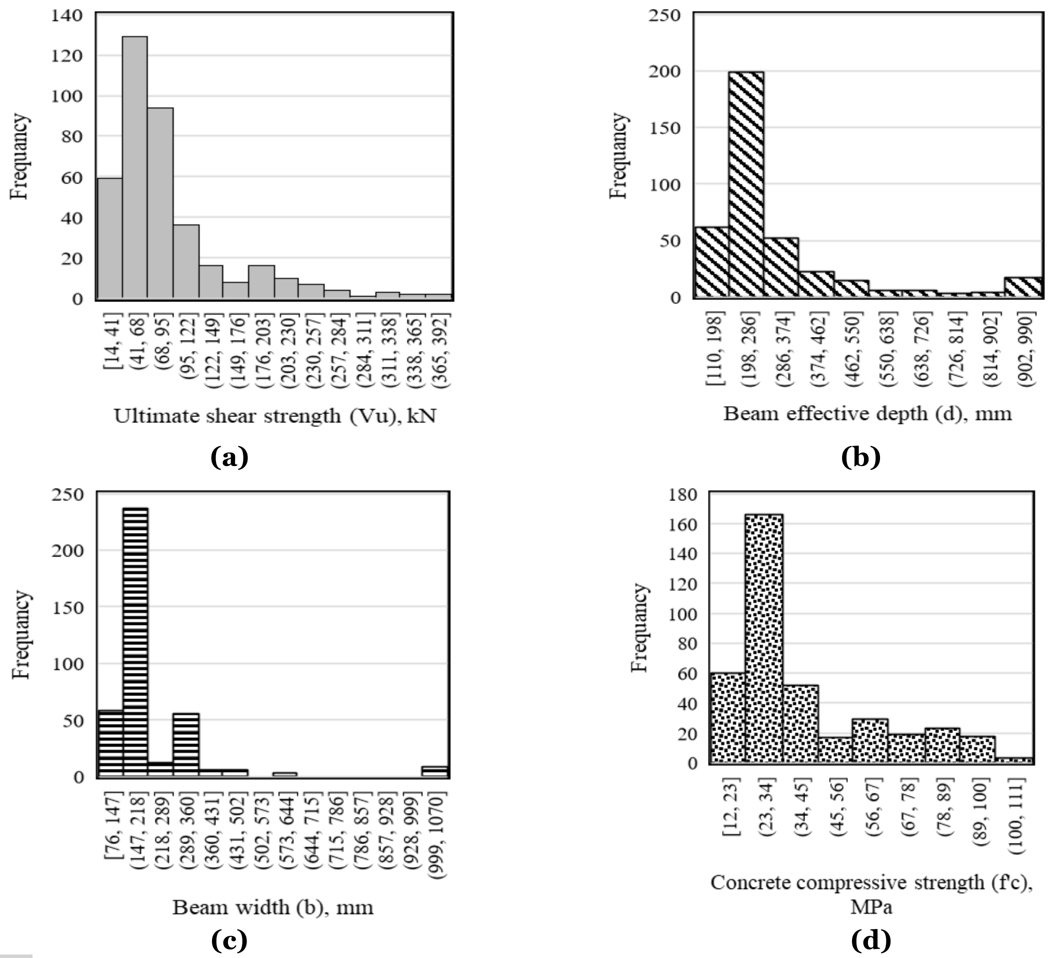

Fig. 1 Frequency Distribution of (a) the Shear Capacity (Vu), (b) Effective Beam Depth, (c) Beam Width, and (d) Concrete Compressive Strength of the Beams Used in this Study Beam.

3.3. Correlation Analysis

To validate the selection of the utilized variables by measuring the nature and strength between each of the variables and the recorded shear capacity, Pearson correlation coefficients (r) were calculated as follows:

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}}$$

r = Pearson correlation coefficient
x_i = values of the x-variable in a sample
$\bar{x}$ = mean of the values of the x-variable
y_i = values of the y-variable in a sample
$\bar{y}$ = mean of the values of the y-variable
The results of the correlation test are as follows:

- Vu vs. d: r = 0.69 (moderate direct correlation)
- Vu vs. b_w: r = 0.73 (strong direct correlation)
- Vu vs. a/d: r = -0.1 (weak indirect correlation)
- Vu vs. A_{sl}: r = 0.85 (strong direct correlation)
- Vu vs. ρ: r = -0.18 (weak indirect correlation)
- Vu vs. f'c: r = 0.2 (weak direct correlation)

The correlation matrix in Fig. 2 revealed meaningful relationships between the independent variables and the dependent variable, Vc (shear capacity). A strong positive correlation was observed between Vu and both the beam width (bw) and the area of longitudinal reinforcement (Asl), indicating that increases in these parameters are strongly associated with higher shear capacity. Similarly, depth (d) exhibited moderate to strong positive correlations with Vu, suggesting its essential roles in influencing beam strength. On the other hand, the shear span-to-depth ratio (a/d) and the longitudinal reinforcement ratio (ρ) showed a notable negative correlation with Vu, aligning with structural mechanics principles, where an increased shear span reduces shear resistance. Variables, such as steel yield strength (fsy), demonstrated relatively weak correlations with Vu, implying limited linear influence in this dataset. Additionally, no excessively high correlations were found among the independent variables themselves, further supported by the low VIF values obtained in the multicollinearity analysis (Table 2), indicating that the model structure is statistically sound, and the predictors are not redundant.

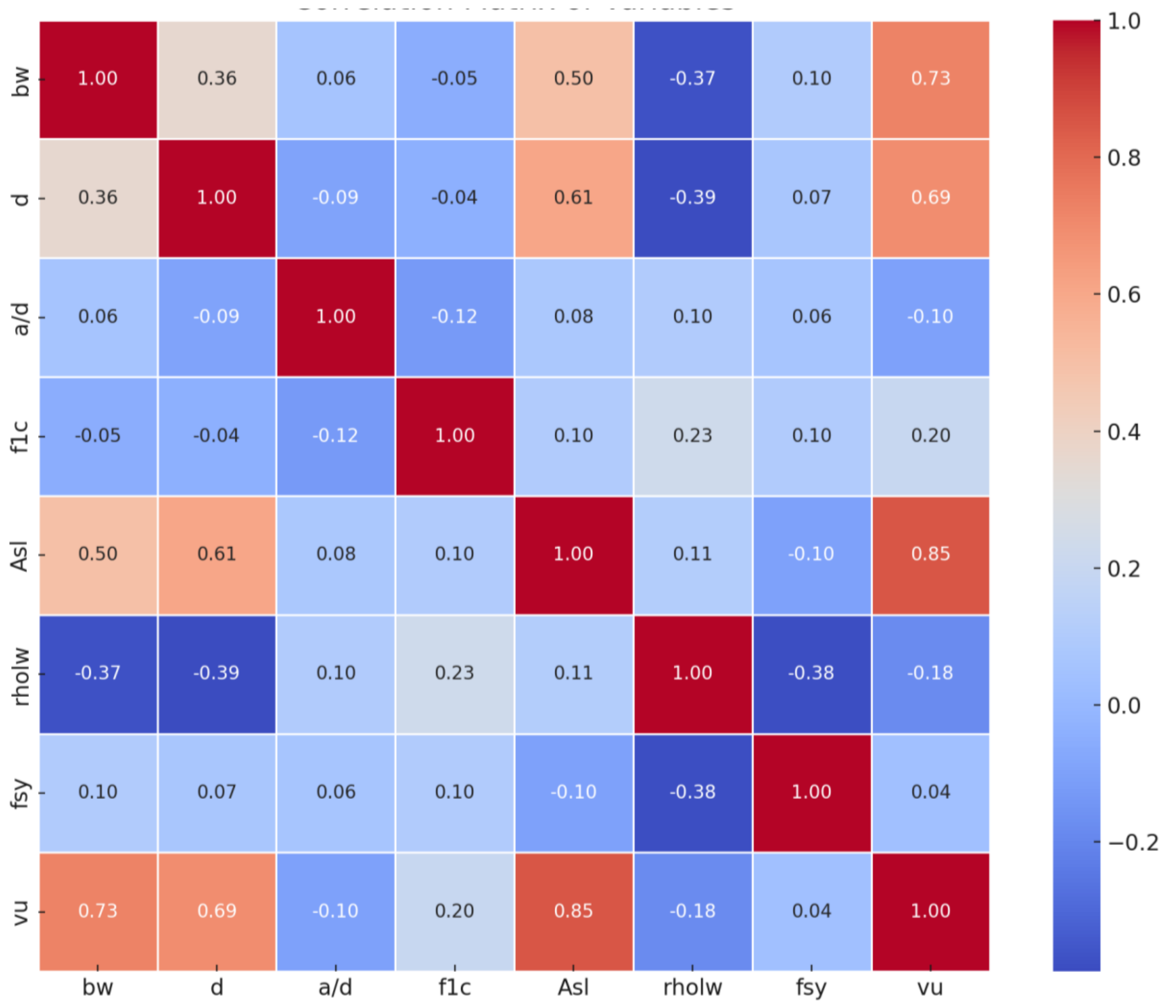

Fig. 2 Correlation Matrix of Structural Parameters Affecting Shear Capacity (Vu).

These correlations affirm the suitability of both parameters for use in regression modeling. Scatter plots of Vu versus b_w, d, A_{sl}, and f'c display upward trends Fig. 3 with a strong regression line for each of the effective depth, beam width, and longitudinal reinforcement area, supporting the hypothesized positive relationships. The nonlinear nature of these trends further justifies the use of polynomial regression in addition to linear models.

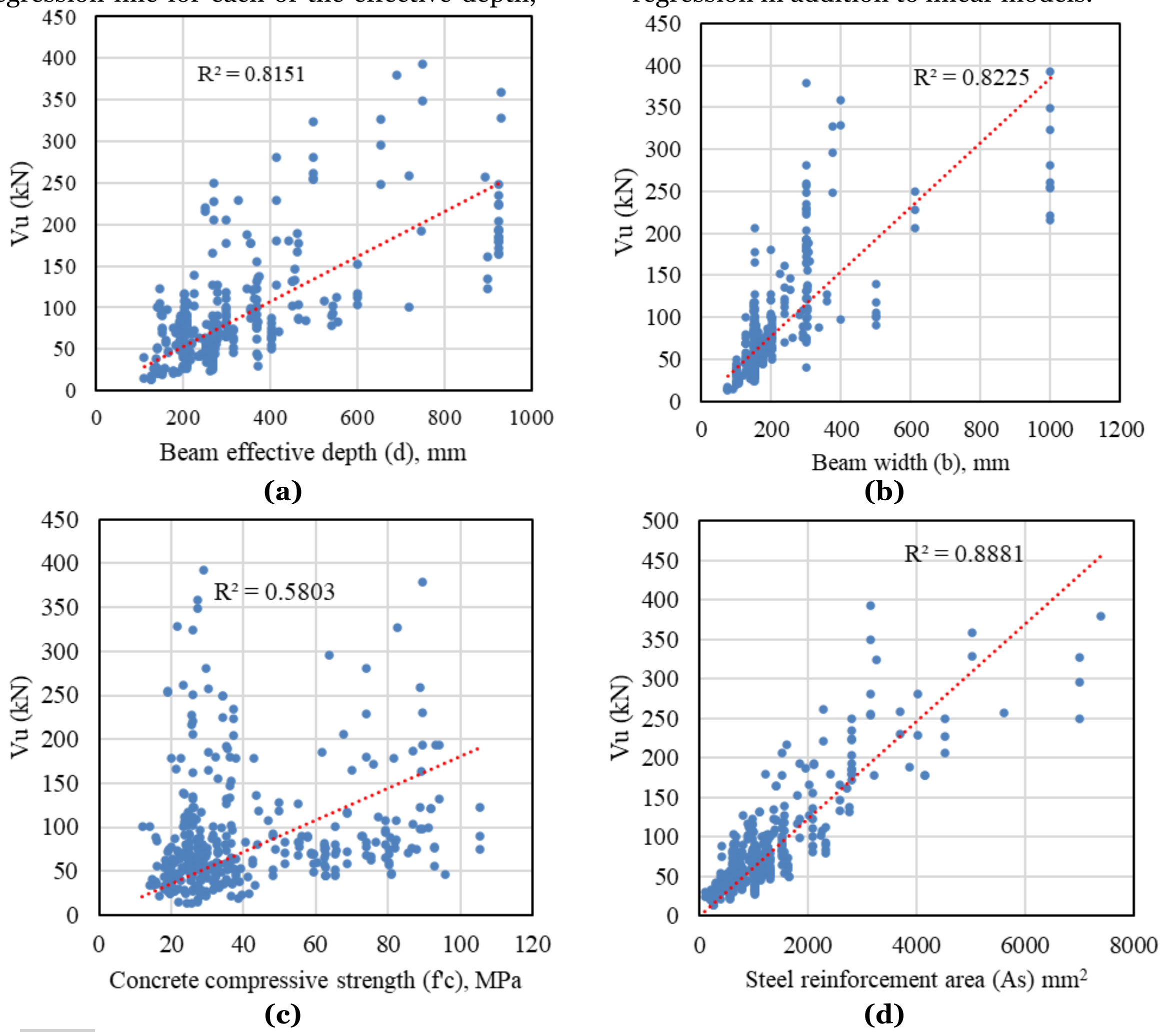

Fig. 3 Scatter Plots and Correlation of Vu Versus (a) Effective Beam Depth, (b) Beam Width, (c) Concrete Compressive Strength, and (d) Steel Reinforcement Area.

3.4.Multicollinearity Check

To ensure the independent variables are not highly collinear, the Variance Inflation Factor (VIF) was computed. Most of the VIF values are well below 5, as shown in Table 3, indicating no significant multicollinearity, which validates the joint inclusion of these variables in the regression models. A VIF below 5 indicates low multicollinearity, where the variable is not strongly correlated with other predictors in the model. Additionally, it provides a reliable estimate, with regression coefficients that are likely stable and interpretable. Finally, a VIF below 5 refers to a good model structure, with no need to remove or combine variables due to correlation issues. In conclusion to all the above variable examinations, the statistical evaluation confirms that all selected parameters are relevant, significant, and independently contribute to predicting the shear capacity. The patterns in the data also support the modeling choices made in the next phase of the research.

Table 2 Variance Inflation Factor (VIF) of the Primary Variables.

Variable	VIF
b_w	1.919932
d	2.657233
a/d	1.093044
f'c	1.161081
A_{sl}	3.225968
ρ	2.543997
fy	1.271095

4.MODELING

This section presents the formulation, development, and implementation of the seven proposed models used to predict the shear capacity (Vc) of reinforced concrete beams without web reinforcement. Each model employs different assumptions and levels of complexity, allowing for a comparative evaluation of their predictive capabilities.

4.1.Linear Regression Model (LR)

The linear regression model assumes a direct linear relationship between shear capacity (Vc) and the input variables, beam width (b_w) and effective depth (d). The general form of the model is given by:

$$Vc=\beta_0+\beta_1 \cdot bw+\beta_2 \cdot d+ \beta_3 \cdot (a/d)+ \beta_4 \cdot \phi+ \beta_5 \cdot fc'+ \beta_6 . Asl+ \beta_7 \cdot \rho +\beta_8 \cdot fy$$

To develop the best-fit model for predicting the shear capacity (Vc) of reinforced concrete beams, a multiple linear regression analysis was employed using all available independent variables in the dataset. This statistical method models the relationship between one dependent variable (Vc) and several independent predictors, including beam width (b_w), effective depth (d), shear span-to-depth ratio (a/d), bar diameter (ϕ), concrete compressive strength (f'c), longitudinal reinforcement area (A_{sl}), reinforcement ratio (ρ), and steel yield strength (fy). The regression was performed using the ordinary least squares (OLS) approach, which estimates the coefficients by minimizing the sum of squared differences between observed and predicted Vc values. The analysis was performed using the scikit-learn library in Python, which provided a robust and efficient implementation of linear regression, allowing for automatic handling of the input features, rapid model fitting, and straightforward extraction of performance metrics, such as the coefficient of determination (R^2). The resulting model, shown below, demonstrated a high level of accuracy, with an R^2 value of 0.9733, indicating that 97.3% of the variability in shear capacity was explained by the selected input parameters.

$$Vc=-10.603+0.198 \cdot b_w+0.085 \cdot d-9.448 \cdot (a/d) +0.119 \cdot \phi+0.481 \cdot f'c+0.027 \cdot A_{sl}+0.444 \cdot \rho+0.009 \cdot fy$$

As shown in Fig. 4 (a), the majority of the data fall within a ±20% error, and the entire dataset is centered around the 45-degree line.

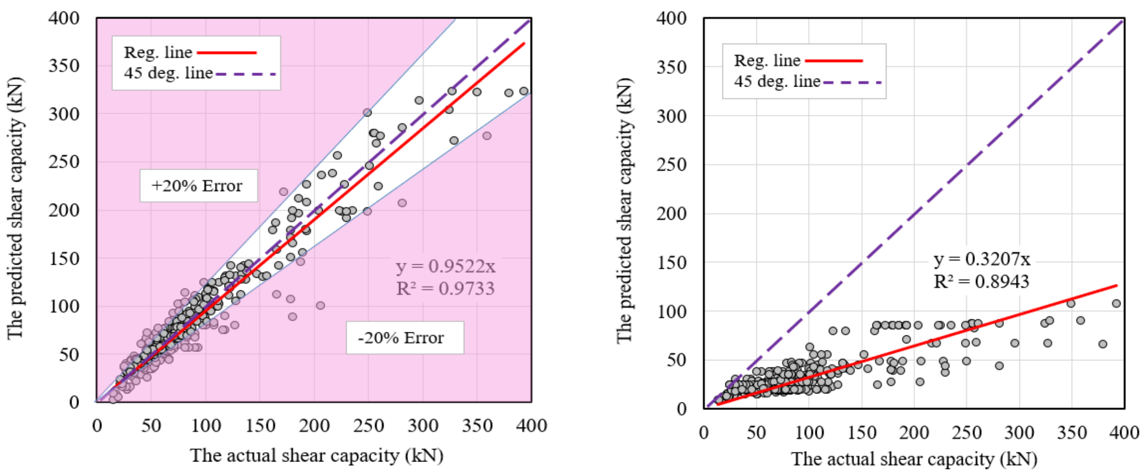

Fig. 4 Comparison between Measured and Predicted Shear Capacity of RC Beam without Web Reinforcement Using Linear Regression Model (LR). (a) Full Model and (b) Simplified Model.

On the other side, the scikit-learn library in Python proposed a simplified version of the model where it uses both the beam's width and effective depth to predict the shear capacity in the following form:-

$$Vc = \beta_0 + \beta_1 \cdot b_w + \beta_2 \cdot d$$

The simplified linear model is simple to interpret and implement. It is best suited for initial assessments and cases where the relationship between inputs and outputs is approximately linear. Although the resulting model, Vc = -4.312 + 0.052·b_w + 0.08·d, produced an acceptable R^2 of 0.8943, all of the predicted capacities were significantly lower than the actual ones, placing all the data below the 45-degree equitizing line, as shown in Fig. 4 (b), resulting in unsafe predictions.

4.2.Stepwise Multivariate Regression (SMR) Using SPSS

Stepwise multivariate regression is a variable selection technique designed to build a statistically optimal regression model by iteratively adding and/or removing predictor variables based on their significance and contribution to the model's performance. Unlike the traditional full regression model, which includes all available predictors regardless of their statistical relevance, stepwise regression refines the model by eliminating redundancy and retaining only variables that meaningfully improve prediction accuracy. This approach balances model complexity and explanatory power, often yielding a more parsimonious model that avoids overfitting. Compared to forward selection, which starts with no variables and adds the most significant ones step-by-step, and backward elimination, which begins with all variables and sequentially removes the least significant, stepwise regression combines both strategies, allowing for both inclusion and exclusion at each step. This dynamic process makes it particularly effective in identifying the most robust subset of predictors, especially when multicollinearity or overlapping effects exist among variables, as observed in the initial full model for predicting ultimate shear strength (Vu) prediction. SPSS software was used to conduct the analysis. It suggests trying the five variables in Table 3 below with their stepwise criteria.

Table 3 The Entered/Removed Variables for the Stepwise Multivariate Regression (SMR)

Model	Variables Entered	Variables Removed	Method
1	A_{sl}	.	Stepwise (Criteria: Probability-of-F-to-enter <= .050, Probability-of-F-to-remove >= .100).
2	b_w	.	Stepwise (Criteria: Probability-of-F-to-enter <= .050, Probability-of-F-to-remove >= .100).
3	d	.	Stepwise (Criteria: Probability-of-F-to-enter <= .050, Probability-of-F-to-remove >= .100).
4	f'c	.	Stepwise (Criteria: Probability-of-F-to-enter <= .050, Probability-of-F-to-remove >= .100).
5	a/d	.	Stepwise (Criteria: Probability-of-F-to-enter <= .050, Probability-of-F-to-remove >= .100).

The standard form of the prediction equation is

$$V_c = A + B.A_{sl} + C.b_w + D.d + E.f'_c + F.\frac{a}{d}$$

and the values of each of the coefficients A, B, C, D, E, and F are listed below in Table 4 based on each one of the five proposed models, as follows:

1- $V_c = 25.073 + 0.049\, A_{sl}$
2- $V_c = -3.352 + 0.037\, A_{sl} + 0.206 b_w$
3- $V_c = -15.78 + 0.027\, A_{sl} + 0.196 b_w + 0.083 d$
4- $V_c = -39.827 + 0.025 A_{sl} + 0.203 b_w + 0.093 d + 0.544 f'_c$
5- $V_c = -4.877 + 0.027\, A_{sl} + 0.201 b_w + 0.086 d + 0.485 f'_c - 9.116 \frac{a}{d}$

Table 4 Equation's Coefficients of the Equations Based on the Stepwise Multivariate Regression (SMR).

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	25.073	3.398		7.379	.000
	A_{sl}	.049	.002	.823	26.608	.000
2	(Constant)	-3.352	2.841		-1.180	.239
	A_{sl}	.037	.001	.618	25.338	.000
	b_w	.206	.011	.453	18.557	.000
3	(Constant)	-15.780	2.476		-6.372	.000
	A_{sl}	.027	.001	.445	18.838	.000
	b_w	.196	.009	.431	21.797	.000
	d	.083	.006	.300	13.413	.000
4	(Constant)	-39.827	3.055		-13.038	.000
	A_{sl}	.025	.001	.411	20.038	.000
	b_w	.203	.008	.447	26.186	.000
	d	.093	.005	.333	17.133	.000
	f'c	.544	.050	.169	10.969	.000
5	(Constant)	-4.877	5.232		-.932	.352
	A_{sl}	.027	.001	.446	23.039	.000
	b_w	.201	.007	.442	28.186	.000
	d	.086	.005	.309	17.075	.000
	f'c	.485	.046	.151	10.511	.000
	a/d	-9.116	1.152	-.114	-7.915	.000

In addition, the quality of each model during the stepwise multivariate regression process is shown in Table 5 based on R^2, and the standard error of estimate, which indicates approximately how much error is made when the predicted value for shear capacity (on the least-squares line) instead of the actual value of shear capacity. The ANOVA test results for each model are also listed.

Table 5 The Quality of the Five Proposed Stepwise Multivariate Regression (SMR) Models.

Model	r	R Square	Adjusted R Square	Std. Error of the Estimate
1	.823[a]	.677	.676	41.17757
2	.917[b]	.840	.839	29.00182
3	.947[c]	.896	.895	23.43970
4	.961[d]	.923	.923	20.13535
5	.967[e]	.936	.935	18.50451

a. Predictors: (Constant), A_{sl} *(Indicating that 67.7% of the variability in shear capacity was explained by the selected input parameters)*
b. Predictors: (Constant), A_{sl}, b_w *(If b_w is added, it will improve to 84%)*
c. Predictors: (Constant), A_{sl}, b_w, d *(If d is added, it will improve to 89.6%)*
d. Predictors: (Constant), A_{sl}, b_w, d , f1c *(If f'c is added, it will improve to 92.3%)*
e. Predictors: (Constant), A_{sl}, b_w, d , f1c , a/d *(If a/d is added, it will improve to 93.6%)*

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1200457.901	1	1200457.901	707.987	.000
	Residual	573110.272	338	1695.593		
	Total	1773568.174	339			
2	Regression	1490115.645	2	745057.822	885.808	.000
	Residual	283452.529	337	841.105		
	Total	1773568.174	339			
3	Regression	1588963.149	3	529654.383	964.025	.000
	Residual	184605.024	336	549.420		
	Total	1773568.174	339			
4	Regression	1637748.346	4	409437.087	1009.878	.000
	Residual	135819.827	335	405.432		
	Total	1773568.174	339			
5	Regression	1659200.978	5	331840.196	969.112	.000
	Residual	114367.195	334	342.417		
	Total	1773568.174	339			

The above results were obtained when SPSS automatically adds or removes predictors based on their statistical contribution to explaining the dependent variable. For example, Model 1 included: A_{sl} with B = 0.049, Beta = 0.823, and $p < .001$, indicating a powerful positive relationship with Vc. The rationale behind selecting A_{sl} first is that it is the most significant predictor. Model 2 included: A_{sl} and b_w with A_{sl}: Beta = 0.618 and b_w: Beta = 0.453, $p < .001$, which indicates that both variables have strong, independent contributions to explaining Vc. Model 3 added the effective depth d with Beta = 0.300, $p < .001$. A_{sl} and b_w retain their significance, although A_{sl}'s Beta drops to 0.445. Model 4 added the compressive strength f'c with Beta = -0.169, $p < .001$, which indicated a negative association with Vc; however, all variables in the model remained significant. Finally, Model 5 added an a/d ratio with Beta = -0.114, $p < .001$, which introduces a slight negative contribution. As a result, A_{sl} and b_w are the strongest predictors, with d and f'c also contributing significantly, and a/d has a weaker but significant negative effect. On the other hand, the excluded variables are ρ, fy, and ϕ due to their insufficient additional explanatory power. The model was built step-by-step and was improved at each stage by adding variables that significantly enhanced prediction, incorporating both positive and negative relationships. It is worth noting that multicollinearity was not present, as all tolerance values were above 0.6.

4.3. Training and Validation

All models were trained using 80% of the dataset and validated on the remaining 20%. Hyperparameters were selected based on cross-validation. For SMR, accuracy metrics were used in place of R^2 and RMSE. The training was performed using the `scikit-learn` library, which provides robust implementations of all four models and ensures reproducibility.

4.4. Visualization of Fits

Regression lines and decision boundaries were plotted for each model to visually assess the fit quality. The resulting model, shown below, demonstrated a high level of accuracy, with an R^2 value ranging from 0.901 for Model 1 to 0.9734 for Model 5, indicating that 97.3% of the variability in shear capacity was explained by the selected input parameters.

As shown in Fig. 5 (e), the majority of the data fall within a ±20% error, and the entire dataset is centered around the 45-degree line.

This section sets the foundation for evaluating how these models perform in practice, which is addressed in the following two sections.

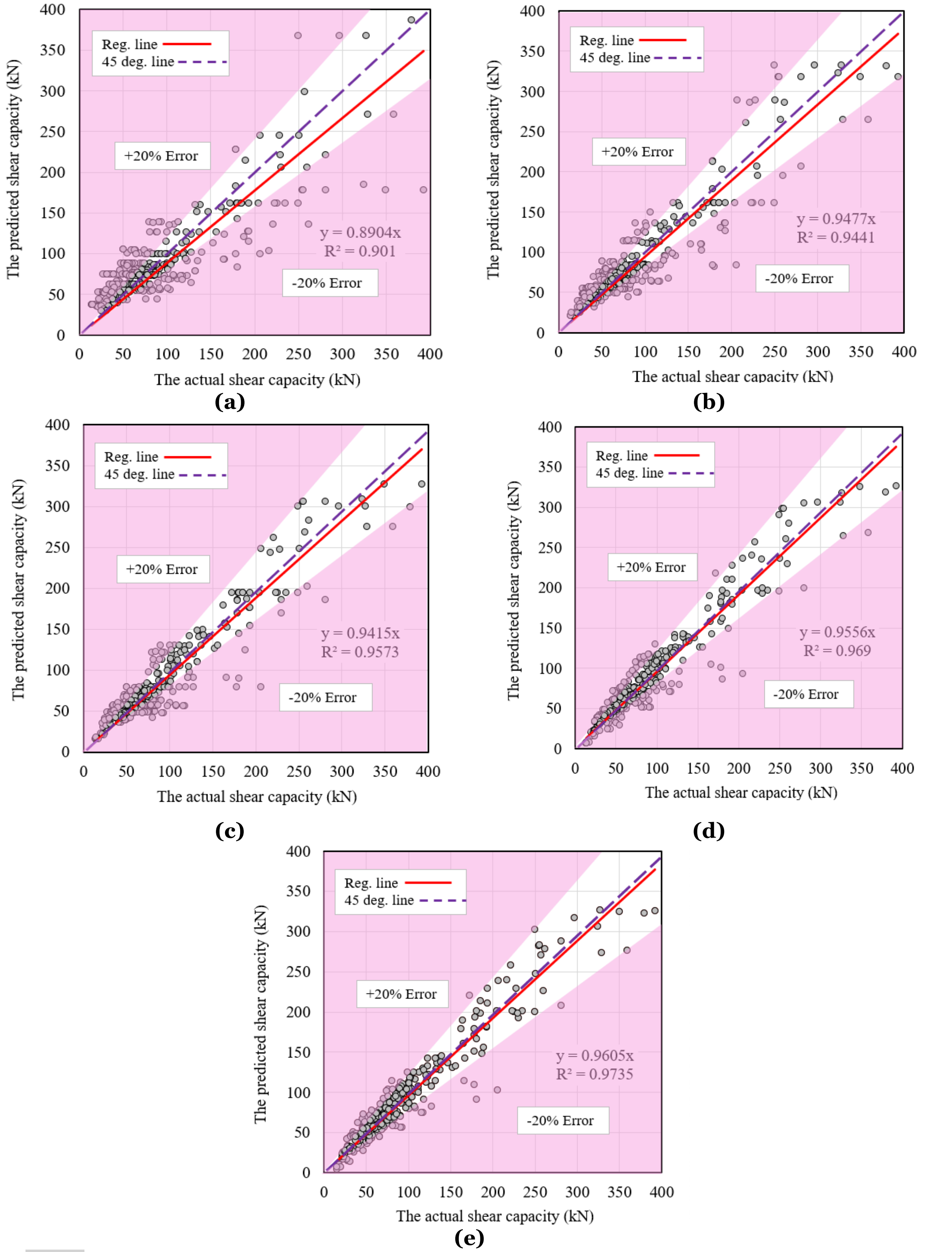

Fig. 5 Comparison between Measured and Predicted Shear Capacity of RC Beam Without Web Reinforcement Using Stepwise Multivariate Regression (SMR) (a) Model 1, (b) Model 2, (c) Model 3, (d) Model 4, and (e) Model 5.

5.ASSESSMENT CRITERIA FOR MODELS

To objectively evaluate the performance of the four predictive models—Linear Regression (LR) and the Stepwise Multivariate Regression (SMR)—a set of widely recognized statistical metrics was employed. These metrics allow for a fair comparison in terms of accuracy, generalization capability, and classification effectiveness.

5.1.Coefficient of Determination (R^2)

The R^2 score measures the proportion of the variance in the dependent variable (Vc) that is predictable from the independent variables. It is defined as:

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

where SS_{res} is the residual sum of squares, and SS_{tot} is the total sum of squares.
An R^2 value closer to 1 indicates a model that explains most of the variability in the data. It applies to both the LR and SMR models.
The calculated Coefficient of Determination for each model is shown in Fig. 6. The measured values that both the full linear model and the Fifth-Stepwise Multivariate Regression model perform better than the other proposed models.

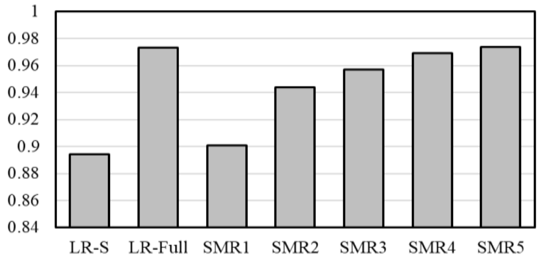

Fig. 6 Comparison between the calculated coefficient of determination (R^2) of each model.

5.2.Root Mean Squared Error (RMSE)

This metric aggregates the magnitudes of prediction errors into a single measure of predictive accuracy (the square root of the mean of squared differences between predicted and actual Vc). A lower RMSE indicates that predictions are, on average, closer to the actual values, where zero would mean a perfect fit, and a smaller RMSE is better. RMSE quantifies the average magnitude of prediction error in units of the dependent variable (MPa). It is computed as:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(y_{pred} - y_{true})^2}{N}}$$

This metric provides insight into the absolute error and is particularly useful when comparing models on datasets with different scales or distributions. Lower RMSE indicates better model performance. Figure 7 compares the measured Root Mean Squared Error for each model, which shows the same previous trend, with both the full linear model and the Fifth-Stepwise Multivariate Regression model being the best among the other proposed models.

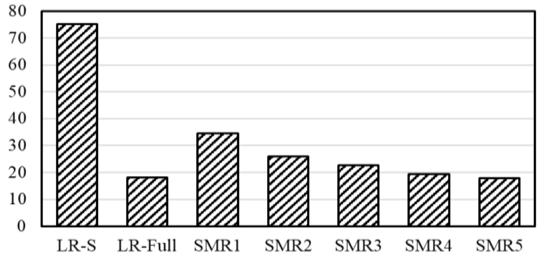

Fig. 7 Comparison between the calculated Root Mean Squared Error (RMSE) of each model.

5.3.Mean Absolute Error (MAE)

The MAE is the average of the absolute differences between predicted and actual Vc values. It provides an easy-to-interpret measure of the typical prediction error in the same units as Vc. A lower MAE means fewer minor errors on average. Although not used as the primary metric in this study, it supports RMSE by providing an error value that is not influenced by outliers. Figure 8 compares the measured Mean Absolute Error for each model, which shows the same previous trend, with both the full linear model and the Fifth-Stepwise Multivariate Regression model being the best among the other proposed models.

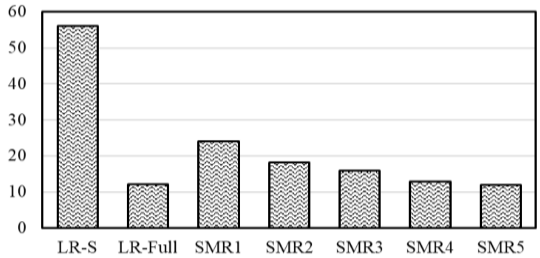

Fig. 8 Comparison between the Calculated Mean Absolute Error (MAE) of Each Model.

5.4.Classification Accuracy

The definition of an "accurate" prediction falls within ±10% of the actual Vc value. This accuracy is calculated as the percentage of predictions within 10% of the actual Vc. In other words, it reflects how often the model's prediction deviates from the actual value by no more than 10% (a higher percentage indicates more predictions meeting this tight tolerance). Accuracy is a simple and interpretable metric, but can be supplemented by confusion matrices and precision-recall scores in more detailed studies. Figure 9 compares the measured Mean Absolute Error for each model, which shows a slightly different trend, with the SMR4 model achieving the best accuracy among the other proposed models.

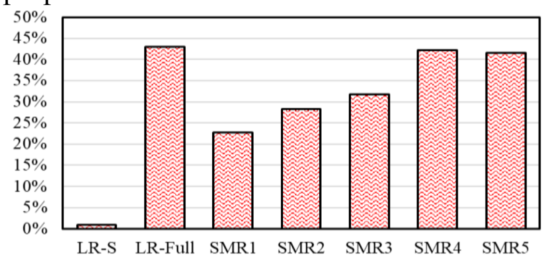

Fig. 9 Comparison between the Calculated Accuracy (±10%) of Each Model.

5.6.Residual Analysis

Graphical residual plots were generated to visualize the distribution and spread of prediction errors. Residuals close to zero and randomly scattered indicate a well-fitted model, which in this case was the SMR5 model. Patterns or curvature in the residuals may suggest model misspecification or nonlinearity.

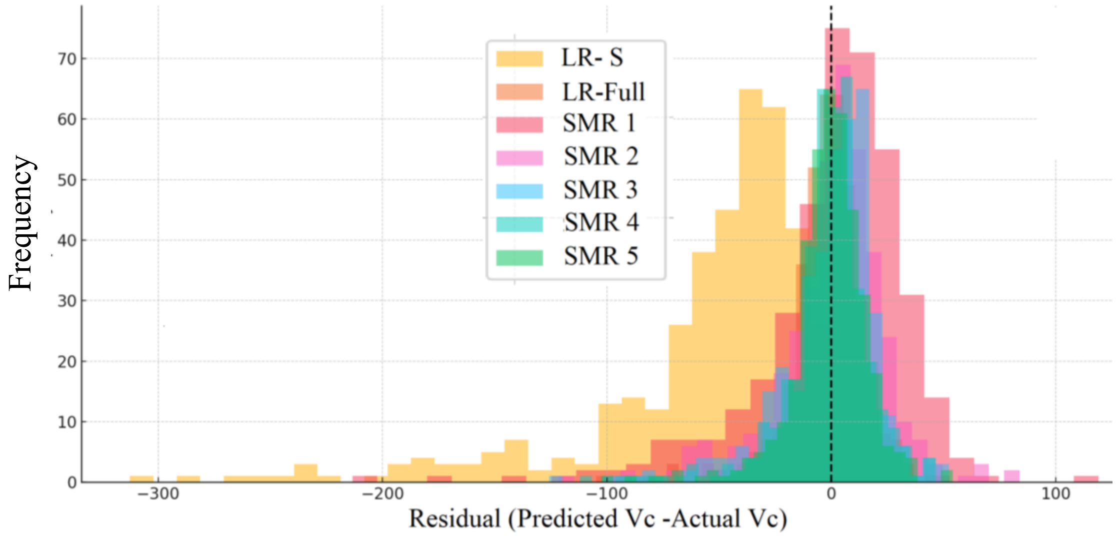

Fig. 10 Residual Distribution for Each Model.

5.7.Summary of Metrics

Each model was evaluated on the test set using the metrics described above. These metrics serve as the basis for a comparative analysis that helps identify the most suitable model for predicting shear capacity under various scenarios. Using the above metrics, the performance for each model was computed. The LR-S model performed worst by a large margin – it has the highest error (RMSE ≈ 75.3) and an extremely low accuracy (only ~1% of its predictions within 10% of the actual Vu), indicating that the LR's predictions deviate significantly from the actual values. LR-Full, SMR4, and SMR5 were the top performers. They achieved much lower errors (RMSE around 17–19) and the highest accuracy (~42% of predictions within 10% of the actual). In particular, LR-Long and SMR5 had the lowest RMSE values (~18 and ~17.8, respectively), while LR-Long slightly outperformed the others in accuracy (≈42.9% within 10%). Overall, these models' predictions are considerably closer to the actual Vu values compared to the rest. The SMR series models show a clear improvement trend from SMR1 to SMR5. Early models (SMR1–SMR3) have moderate errors (RMSE ~22–34) and relatively low accuracy (23–32%); however, the later models, i.e., SMR4 and SMR5, achieve much better accuracy (~42%) and lower errors. This result suggests that the higher-index SMR models provide more accurate Vc predictions (SMR5 being the best in terms of lowest RMSE). It is noteworthy that LR-Full performs almost as well as the best SMR models, indicating that this longer-form regression captures the trend in Vc more effectively than the simplified LR-S. All other models (SMR1–SMR3) fall in between these extremes, with improved performance as the model complexity increases. Overall, LR-Full, SMR4, and SMR5 stand out for their higher accuracy and lower error, making them the most reliable for predicting Vu in this dataset, whereas the simplified LR-S model considerably underestimates Vc (resulting in significant errors and poor accuracy). The choice of model thus significantly impacted the prediction quality, with the more refined models (especially SMR5)

7.CONCLUSIONS

This study proposed and evaluated seven data-driven models, utilizing both Linear Regression (LR) and the Stepwise Multivariate Regression (SMR), to predict the shear capacity (Vc) of reinforced concrete beams without web reinforcement. Using a curated database of 398 experimentally tested specimens, the study demonstrated the potential of multi-scale statistical modeling for structural assessment and design applications. The models developed in this research can serve as powerful tools for structural engineers, code developers, and researchers. The main conclusions of the study are as follows:

1- Each of the longitudinal reinforcement area, beam width (b_w) and effective depth (d), was shown to have a significant influence on shear capacity, with effective depth exhibiting the strongest correlations of 0.85, 0.73, and 0.69, respectively.
2- The full linear model, LR-Full, achieved strong predictive capability (R^2 = 0.9733), suitable for reliable estimations of conservative assessments.
3- The SMR models showed promising improvement from SMR1 to SMR5, with RMSE decreasing from 34.5 MPa to 17.8 MPa and residuals becoming increasingly centered around zero. This result proves the effectiveness of stepwise refinement in capturing complex structural behavior and reducing prediction error beyond what linear models could achieve.

4- The simplified LR model severely underperformed, with very high RMSE and nearly zero accuracy. Even the improved LR-full model could not outperform the top SMR models in all metrics. However, it came close in accuracy, indicating that stepwise feature engineering and model refinement capture structural behavior more effectively than general linear models.
5- The SMR5 model can be embedded in structural design software for precise shear capacity estimation. The LR-Full model remains relevant for preliminary design and educational purposes due to its simplicity and ease of use.

7.1.Recommendations for Future Work

Despite their excellent predictive image, the model is developed based on an experimental design with certain specific settings, and not for all RC beams. They are conditioned on the test inputs, and overextending their use beyond the region of validity may result in unsafe decisions. Future studies may consider the following:

1- Extending the models to include material properties, load types, and environmental effects.
2- Comparing statistical models with machine learning techniques, such as decision trees, random forests, and neural networks.
3- Creating software plugins or mobile apps based on these models for industry-wide adoption.
4- Validating model predictions using full-scale structural tests or field data.

Appendix A: Table (1) Experimental Dataset Structure

The dataset used in this study comprises 398 experimentally tested reinforced concrete beams without web reinforcement. Each entry in the dataset includes the following parameters:
- Beam width (bw) [mm]
- Effective depth (h) [mm]
- Shear span (a) [mm]
- Shear span-to-depth ratio (a/d)
- Longitudinal reinforcement ratio (ρ)
- Concrete compressive strength (fc) [MPa]
- Measured shear capacity (Vc) [MPa]

The dataset was verified, cleaned, and normalized before analysis.

Authors	b_w	d	a/d	ϕ	f'c	A_{sl}	ρ	fy	Vc
Adebar, Collins (1996)	360	278	2.88	19.0	49.9	1570	1.57	536	128
Adebar, Collins (1996)	360	278	2.88	19.0	49.9	1570	1.57	536	119
Adebar, Collins (1996)	290	278	2.88	19.0	46.8	1570	1.95	536	108
Adebar, Collins (1996)	290	278	2.88	19.0	43.9	1570	1.95	536	81
Adebar, Collins (1996)	290	178	4.49	19.0	48.9	1570	3.04	536	75
Adebar, Collins (1996)	290	278	2.88	19.0	56.0	800	0.99	536	90
Ahmad, Kahloo (1986)	127	203	4.00	12.7	59.3	1013	3.93	414	58
Ahmad, Kahloo (1986)	127	203	3.00	12.7	59.3	1013	3.93	414	69
Ahmad, Kahloo (1986)	127	203	2.70	12.7	59.3	1013	3.93	414	69
Ahmad, Kahloo (1986)	127	208	3.00	12.7	59.3	467	1.77	414	49
Ahmad, Kahloo (1986)	127	202	4.00	12.7	65.3	1289	5.03	414	51
Ahmad, Kahloo (1986)	127	202	3.00	12.7	65.3	1289	5.03	414	69
Ahmad, Kahloo (1986)	127	202	2.70	12.7	65.3	1289	5.03	414	100
Ahmad, Kahloo (1986)	127	208	4.00	12.7	65.3	594	2.25	414	44
Ahmad, Kahloo (1986)	127	208	3.00	12.7	65.3	594	2.25	414	47
Ahmad, Kahloo (1986)	127	208	2.70	12.7	65.3	594	2.25	414	80
Ahmad, Kahloo (1986)	127	184	4.00	12.7	62.7	1552	6.64	414	54
Ahmad, Kahloo (1986)	127	184	3.00	12.7	62.7	1552	6.64	414	76
Ahmad, Kahloo (1986)	127	184	2.70	12.7	62.7	1552	6.64	414	69
Ahmad, Kahloo (1986)	127	207	4.00	12.7	62.7	855	3.26	414	45
Ahmad, Kahloo (1986)	127	207	3.00	12.7	62.7	855	3.26	414	44
Ahmad, Kahloo (1986)	127	207	2.70	12.7	62.7	855	3.26	414	45
Al-Alusi (1957)	76	127	4.50	6.4	24.2	253	2.62	366	14
Al-Alusi (1957)	76	127	4.00	6.4	27.2	253	2.62	366	15
Al-Alusi (1957)	76	127	3.40	6.4	27.2	253	2.62	366	17
Al-Alusi (1957)	76	127	4.50	6.4	25.6	253	2.62	366	14
Angelakos, Bentz, Collins ()	300	925	2.92	10.0	20.0	2800	1.01	550	179
Angelakos, Bentz, Collins ()	300	925	2.92	10.0	30.4	2800	1.01	550	185
Angelakos, Bentz, Collins ()	300	925	2.92	10.0	36.1	2800	1.01	550	180
Angelakos, Bentz, Collins ()	300	925	2.92	10.0	61.8	2800	1.01	550	185
Angelakos, Bentz, Collins ()	300	925	2.92	10.0	76.0	2800	1.01	550	172
Angelakos, Bentz, Collins ()	300	895	3.02	10.0	30.4	5600	2.09	550	257
Angelakos, Bentz, Collins ()	300	925	2.92	10.0	30.4	1400	0.50	550	165
Aster; Koch (1974)	1000	250	3.68	30.0	25.6	1600	0.64	554	216
Aster; Koch (1974)	1000	250	3.68	30.0	26.0	2280	0.91	535	221
Aster; Koch (1974)	1000	500	5.50	30.0	29.5	3140	0.63	536	281
Aster; Koch (1974)	1000	500	5.50	30.0	18.9	3140	0.63	536	254
Aster; Koch (1974)	1000	500	5.50	30.0	19.0	3140	0.63	536	255

Authors	b_w	d	a/d	ϕ	f'c	A_{sl}	ρ	fy	Vc
Aster; Koch (1974)	1000	500	3.65	30.0	23.3	2280	0.46	535	261
Aster; Koch (1974)	1000	500	3.65	30.0	26.0	3260	0.65	535	324
Aster; Koch (1974)	1000	750	3.67	30.0	28.8	3140	0.42	536	392
Aster; Koch (1974)	1000	750	3.67	30.0	27.3	3140	0.42	536	349
Bhal (1968)	240	300	3.00	30.0	22.0	904	1.26	434	70
Bhal (1968)	240	600	3.00	30.0	28.1	1808	1.26	434	117
Bhal (1968)	240	900	3.00	30.0	26.1	2712	1.26	434	162
Bhal (1968)	240	600	3.00	30.0	25.3	904	0.63	434	104
Bhal (1968)	240	600	3.00	30.0	23.5	904	0.63	430	112
Bhal (1968)	240	900	3.00	30.0	25.9	1356	0.63	434	135
Bhal (1968)	240	900	3.00	30.0	26.3	1356	0.63	430	123
Bresler, Scordelis (1963)	310	461	3.80	19.1	21.4	2579	1.81	555	167
Bresler, Scordelis (1963)	305	466	4.74	19.1	22.5	3224	2.27	555	178
Bresler, Scordelis (1963)	307	462	6.77	19.1	35.7	3868	2.73	552	189
Cederwall K., Hedman O., Losberg A. (1974)	135	234	3.42		27.8	339	1.07	818	41
Chana (1981)	203	356	3.00	20.0	37.0	1257	1.74	478	96
Chana (1981)	203	356	3.00	10.0	31.2	1257	1.74	478	87
Chana (1981)	203	356	3.00	20.0	33.9	1257	1.74	478	99
Collins, Kuchma (1999)	300	925	2.92	10.0	34.2	2800	1.01	550	225
Collins, Kuchma (1999)	300	925	2.92	10.0	93.1	2800	1.01	550	193
Collins, Kuchma (1999)	300	925	2.92	10.0	37.1	2800	1.01	550	204
Collins, Kuchma (1999)	300	925	2.92	10.0	37.1	2800	1.01	483	223
Collins, Kuchma (1999)	300	925	2.92	10.0	34.2	2800	1.01	550	249
Collins, Kuchma (1999)	300	925	2.92	10.0	37.1	2800	1.01	483	235
Diaz de Cossio, Siess (1960)	152	254	3.00	25.4	29.9	380	0.98	469	42
Diaz de Cossio, Siess (1960)	152	254	4.00	25.4	18.5	380	0.98	452	34
Diaz de Cossio, Siess (1960)	152	254	3.00	25.4	25.4	1289	3.33	314	59
Diaz de Cossio, Siess (1960)	152	254	4.00	25.4	21.0	1289	3.33	393	47
Diaz de Cossio, Siess (1960)	152	254	5.00	25.4	26.1	1289	3.33	364	55
Elzanaty, Nilson, Slate (1986)	178	270	4.00	12.7	62.2	570	1.19	434	57
Elzanaty, Nilson, Slate (1986)	178	268	4.00	12.7	62.2	1164	2.44	434	66
Elzanaty, Nilson, Slate (1986)	178	267	4.00	12.7	62.2	1520	3.21	434	75
Elzanaty, Nilson, Slate (1986)	178	268	4.00	12.7	75.3	776	1.63	434	62
Elzanaty, Nilson, Slate (1986)	178	268	4.00	12.7	75.3	1164	2.44	434	66
Elzanaty, Nilson, Slate (1986)	178	268	6.00	12.7	60.3	1164	2.44	434	60
Elzanaty, Nilson, Slate (1986)	178	270	4.00	12.7	19.7	570	1.19	434	44
Elzanaty, Nilson, Slate (1986)	178	268	4.00	12.7	19.7	1164	2.44	434	53
Elzanaty, Nilson, Slate (1986)	178	273	4.00	12.7	38.0	451	0.93	434	45
Elzanaty, Nilson, Slate (1986)	178	270	4.00	12.7	38.0	570	1.19	434	48
Elzanaty, Nilson, Slate (1986)	178	268	4.00	12.7	38.0	1164	2.44	434	63
Feldman, Siess (1955)	152	252	3.02	25.4	34.9	1290	3.35	283	80
Feldman, Siess (1955)	152	252	4.02	25.4	26.6	1290	3.35	310	53
Feldman, Siess (1955)	152	252	5.03	25.4	24.5	1290	3.35	303	51
Feldman, Siess (1955)	152	252	6.04	25.4	26.5	1290	3.35	331	51
Ferguson P.M. (1956)	101	189	3.23	6.4	27.8	396	2.08	310	22
Ferguson, Thompson (1953)	102	210	3.39	6.4	28.2	1013	4.76	276	29
Ferguson, Thompson (1953)	102	210	3.39	6.4	25.9	1013	4.76	276	27
Ferguson, Thompson (1953)	102	210	3.39	6.4	33.3	1013	4.76	276	34
Ferguson, Thompson (1953)	102	210	3.39	6.4	33.2	1013	4.76	276	32
Ferguson, Thompson (1953)	102	210	3.39	6.4	43.1	1013	4.76	276	34
Ferguson, Thompson (1953)	102	210	3.39	6.4	36.8	1013	4.76	276	36
Ferguson, Thompson (1953)	178	210	3.39	6.4	29.7	1013	2.72	276	49
Ferguson, Thompson (1953)	178	210	3.39	6.4	28.1	1013	2.72	276	52
Ferguson, Thompson (1953)	108	178	4.00	15.9	19.7	570	2.97	276	24
Ferguson, Thompson (1953)	108	178	4.00	15.9	19.6	570	2.97	276	24
Ferguson, Thompson (1953)	108	178	4.00	15.9	16.6	570	2.97	276	21
Ferguson, Thompson (1953)	102	210	3.39	6.4	33.9	1013	4.76	276	35
Ferguson, Thompson (1953)	102	210	3.39	6.4	31.8	1013	4.76	276	32
Ferguson, Thompson (1953)	102	210	3.39	6.4	38.0	1013	4.76	276	39
Ferguson, Thompson (1953)	102	210	3.39	6.4	41.2	1013	4.76	276	44
Ferguson, Thompson (1953)	102	210	3.39	6.4	39.0	1013	4.76	276	38
Ferguson, Thompson (1953)	102	210	3.39	6.4	31.8	1013	4.76	276	50
Ferguson, Thompson (1953)	102	210	3.39	6.4	31.8	1013	4.76	276	39
Ferguson, Thompson (1953)	108	159	4.48	15.9	20.6	570	3.33	276	27
Ferguson, Thompson (1953)	108	159	4.48	15.9	21.1	570	3.33	276	27
Grimm (1997)	300	153	3.73	16.0	85.6	616	1.34	660	70
Grimm (1997)	300	152	3.75	16.0	86.6	1010	2.21	517	76
Grimm (1997)	300	146	3.90	16.0	89.0	1850	4.22	487	99
Grimm (1997)	300	348	3.53	16.0	86.7	1960	1.88	469	187
Grimm (1997)	300	348	3.53	16.0	89.0	982	0.94	469	123
Grimm (1997)	300	328	3.75	16.0	89.4	3700	3.76	487	230
Grimm (1997)	300	718	3.66	16.0	89.0	3700	1.72	487	259
Grimm (1997)	300	746	3.53	16.0	89.7	1850	0.83	487	193
Grimm (1997)	300	690	3.81	16.0	89.4	7390	3.57	487	379
Grimm (1997)	300	153	3.73	16.0	105.4	616	1.34	660	74
Grimm (1997)	300	152	3.75	16.0	105.4	1010	2.21	517	90

Authors	b_w	d	a/d	ϕ	f'c	A_{sl}	ρ	fy	Vc
Grimm (1997)	300	146	3.90	16.0	105.4	1850	4.22	487	122
Hallgren (1994)	163	192	3.65	18.0	81.9	678	2.17	630	83
Hallgren (1994)	158	194	3.61	18.0	81.9	678	2.21	630	77
Hallgren (1994)	158	193	3.63	18.0	80.4	678	2.22	630	76
Hallgren (1994)	155	196	3.57	18.0	58.7	678	2.23	443	70
Hallgren (1994)	156	195	3.59	18.0	65.6	678	2.23	443	74
Hallgren (1994)	157	191	3.66	18.0	42.7	678	2.26	630	59
Hallgren (1994)	155	194	3.61	18.0	42.7	678	2.25	630	63
Hallgren (1994)	155	194	3.61	18.0	80.4	678	2.25	630	69
Hallgren (1994)	156	193	3.63	18.0	58.7	678	2.25	443	71
Hallgren (1994)	156	194	3.61	18.0	57.8	1206	3.98	494	89
Hallgren (1994)	156	195	3.59	18.0	57.8	1206	3.96	494	90
Hallgren (1994)	156	195	3.59	18.0	62.4	1206	3.96	494	82
Hallgren (1994)	155	195	3.59	18.0	62.4	1206	3.99	494	79
Hallgren (1994)	156	196	3.57	18.0	55.4	1206	3.94	494	78
Hallgren (1994)	150	196	3.57	18.0	55.4	1206	4.10	494	83
Hallgren (1994)	156	191	3.66	18.0	31.2	678	2.28	651	56
Hallgren (1994)	156	194	3.61	18.0	31.2	678	2.24	651	54
Hallgren (1994)	156	192	3.65	18.0	29.5	678	2.26	651	49
Hallgren (1994)	157	193	3.63	18.0	29.5	667	2.20	651	54
Hallgren (1996)	262	208	2.64	18.0	87.8	402	0.74	632	76
Hallgren (1996)	283	211	2.61	18.0	86.7	628	1.05	604	104
Hallgren (1996)	337	208	2.64	18.0	80.8	402	0.57	630	89
Hamadi (1976)	100	370	3.39	20.0	28.8	629	1.70	400	45
Hamadi (1976)	100	372	3.37	20.0	22.3	402	1.08	460	41
Hamadi (1976)	100	372	5.90	20.0	20.9	402	1.08	800	30
Hanson (1958)	152	267	2.48		24.2	1013	2.49	333	80
Hanson (1958)	152	267	2.48		26.3	1013	2.49	333	58
Hanson (1958)	152	267	2.48		35.2	1013	2.49	333	90
Hanson (1958)	152	267	2.48		55.1	2027	4.99	333	127
Hanson (1958)	152	267	2.48		70.0	2027	4.99	333	165
Hanson (1961)	152	267	4.95		19.9	507	1.25	611	34
Hanson (1961)	152	267	4.95		29.4	507	1.25	611	43
Hanson (1961)	152	267	4.95		28.2	507	1.25	611	40
Hanson (1961)	152	267	4.95		29.3	1029	2.53	637	52
Hanson (1961)	152	267	2.48		28.6	507	1.25	334	46
Islam, Pam, Kwan (1998)	150	203	3.94	10.0	79.1	982	3.22	532	65
Islam, Pam, Kwan (1998)	150	203	2.96	10.0	79.1	982	3.22	532	108
Islam, Pam, Kwan (1998)	150	203	2.96	10.0	79.1	982	3.22	532	97
Islam, Pam, Kwan (1998)	150	203	3.94	10.0	79.1	982	3.22	532	81
Islam, Pam, Kwan (1998)	150	203	3.94	10.0	68.6	982	3.22	532	58
Islam, Pam, Kwan (1998)	150	203	2.96	10.0	68.6	982	3.22	532	117
Islam, Pam, Kwan (1998)	150	203	2.96	10.0	68.6	982	3.22	532	115
Islam, Pam, Kwan (1998)	150	203	3.94	10.0	68.6	982	3.22	532	72
Islam, Pam, Kwan (1998)	150	207	3.86	10.0	48.3	628	2.02	554	46
Islam, Pam, Kwan (1998)	150	207	2.90	10.0	48.3	628	2.02	554	92
Islam, Pam, Kwan (1998)	150	207	2.90	10.0	48.3	628	2.02	554	90
Islam, Pam, Kwan (1998)	150	207	3.86	10.0	48.3	628	2.02	554	52
Islam, Pam, Kwan (1998)	150	205	3.90	10.0	32.7	982	3.19	320	55
Islam, Pam, Kwan (1998)	150	205	2.93	10.0	32.7	982	3.19	320	85
Islam, Pam, Kwan (1998)	150	205	2.93	10.0	32.7	982	3.19	320	81
Islam, Pam, Kwan (1998)	150	207	3.86	10.0	25.3	628	2.02	350	48
Islam, Pam, Kwan (1998)	150	207	2.90	10.0	25.3	628	2.02	350	57
Kani (1967)	154	543	4.00	19.1	24.9	2323	2.77	352	93
Kani (1967)	156	541	8.03	19.1	24.4	2323	2.75	352	79
Kani (1967)	156	541	6.01	19.1	25.1	2323	2.75	352	91
Kani (1967)	153	556	6.84	19.1	24.8	2316	2.72	381	84
Kani (1967)	152	524	3.11	19.1	25.9	2265	2.84	367	108
Kani (1967)	155	544	2.99	19.1	26.0	2245	2.66	373	102
Kani (1967)	611	271	5.02	19.1	25.6	4510	2.73	377	228
Kani (1967)	612	271	4.01	19.1	25.8	4510	2.72	377	206
Kani (1967)	612	270	3.02	19.1	25.8	4510	2.73	377	250
Kani (1967)	152	138	3.93	19.1	23.6	568	2.69	392	29
Kani (1967)	151	133	5.09	19.1	23.5	568	2.82	392	27
Kani (1967)	153	274	5.93	19.1	26.1	1161	2.76	343	51
Kani (1967)	151	271	4.00	19.1	26.1	1161	2.84	342	55
Kani (1967)	153	275	3.94	19.1	24.0	1161	2.76	335	56
Kani (1967)	156	271	3.00	19.1	26.1	1161	2.74	343	65
Kani (1967)	152	276	2.95	19.1	25.9	1129	2.68	366	62
Kani (1967)	153	137	3.46	19.1	25.9	561	2.67	403	28
Kani (1967)	152	138	3.44	19.1	25.9	561	2.66	417	29
Kani (1967)	155	139	2.93	19.1	25.4	568	2.64	392	39
Kani (1967)	154	269	6.06	19.1	26.1	1123	2.70	364	51
Kani (1967)	152	270	7.03	19.1	26.1	1123	2.73	369	46
Kani (1967)	152	141	2.41	19.1	25.9	561	2.61	381	51
Kani (1967)	154	140	2.67	19.1	25.3	568	2.63	392	50

Authors	b_w	d	a/d	ϕ	f'c	A_{sl}	ρ	fy	Vc
Kani (1967)	150	552	2.46	19.1	25.6	2329	2.82	374	112
Kani (1967)	153	275	2.47	19.1	24.0	1161	2.75	338	73
Kani (1967)	153	275	2.47	19.1	24.9	1129	2.68	366	76
Kani (1967)	152	272	2.50	19.1	24.9	1129	2.73	366	77
Krefeld, Thurston (1966)	152	314	2.71		28.7	1635	3.42	401	73
Krefeld, Thurston (1966)	152	238	3.58		28.6	1635	4.51	401	64
Krefeld, Thurston (1966)	152	316	2.69		18.3	1289	2.68	478	63
Krefeld, Thurston (1966)	152	316	2.69		18.9	1289	2.68	478	72
Krefeld, Thurston (1966)	152	316	2.69		21.5	1289	2.68	478	73
Krefeld, Thurston (1966)	152	316	2.69		21.0	1289	2.68	478	60
Krefeld, Thurston (1966)	152	240	3.55		21.1	645	1.76	478	42
Krefeld, Thurston (1966)	152	243	3.50		20.9	776	2.10	408	44
Krefeld, Thurston (1966)	152	256	4.52		19.8	776	1.99	386	44
Krefeld, Thurston (1966)	152	256	5.72		19.5	776	1.99	386	36
Krefeld, Thurston (1966)	152	256	3.33		32.8	776	1.99	386	56
Krefeld, Thurston (1966)	152	254	3.35		27.7	1013	2.62	401	58
Krefeld, Thurston (1966)	152	252	3.37		31.2	1289	3.35	378	57
Krefeld, Thurston (1966)	152	250	3.40		32.7	1635	4.28	368	60
Krefeld, Thurston (1966)	152	256	4.52		30.3	776	1.99	386	53
Krefeld, Thurston (1966)	152	254	4.55		29.0	1013	2.62	401	54
Krefeld, Thurston (1966)	152	252	4.58		31.2	1289	3.35	378	54
Krefeld, Thurston (1966)	152	250	4.61		32.4	1635	4.28	368	59
Krefeld, Thurston (1966)	152	254	5.75		36.5	1013	2.62	401	53
Krefeld, Thurston (1966)	152	252	5.78		35.6	1289	3.35	378	57
Krefeld, Thurston (1966)	152	250	5.83		36.5	1635	4.28	368	63
Krefeld, Thurston (1966)	203	483	3.03		15.9	1520	1.55	401	85
Krefeld, Thurston (1966)	152	254	5.75		33.9	1013	2.62	369	49
Krefeld, Thurston (1966)	152	254	5.75		37.1	1013	2.62	368	53
Krefeld, Thurston (1966)	254	456	3.87		36.4	2579	2.23	367	147
Krefeld, Thurston (1966)	254	456	3.87		36.4	2579	2.23	366	134
Krefeld, Thurston (1966)	152	316	2.89		19.1	645	1.34	386	46
Krefeld, Thurston (1966)	152	316	2.89		19.7	645	1.34	386	52
Kulkarni, Shah (1998)	102	152	5.00	9.5	38.5	214	1.38	518	20
Kulkarni, Shah (1998)	102	152	4.00	9.5	39.6	214	1.38	518	23
Kulkarni, Shah (1998)	102	152	3.50	9.5	41.4	214	1.38	518	24
Küng (1985)	140	200	2.50	30.0	18.8	157	0.56	504	26
Küng (1985)	140	200	2.50	30.0	17.9	226	0.81	497	30
Küng (1985)	140	200	2.50	30.0	17.9	308	1.10	492	43
Küng (1985)	140	200	2.50	30.0	17.9	509	1.82	507	54
Küng (1985)	140	200	2.50	30.0	19.1	308	1.10	492	40
Lambotte, Taerwe (1990)	200	415	3.01		35.3	804	0.97	545	127
Lambotte, Taerwe (1990)	200	415	3.01		32.3	1206	1.45	545	180
Laupa, Siess (1953)	152	269	4.82	25.4	25.6	852	2.08	284	42
Laupa, Siess (1953)	152	265	4.89	25.4	30.7	1019	2.52	410	53
Laupa, Siess (1953)	152	263	4.92	25.4	29.3	1290	3.21	309	56
Laupa, Siess (1953)	152	262	4.95	25.4	28.4	1639	4.11	315	50
Laupa, Siess (1953)	152	267	4.85	25.4	14.0	774	1.90	328	34
Laupa, Siess (1953)	152	262	4.95	25.4	24.9	1639	4.11	304	50
Leonhardt (1962)	502	148	3.31	30.0	23.6	679	0.91	427	91
Leonhardt (1962)	500	146	3.36	30.0	23.6	1357	1.86	427	106
Leonhardt (1962)	190	270	3.00	30.0	27.4	1062	2.07	465	60
Leonhardt (1962)	190	270	3.00	30.0	27.4	1062	2.07	465	77
Leonhardt (1962)	190	270	4.07	30.0	27.4	1062	2.07	465	61
Leonhardt (1962)	190	270	4.07	30.0	27.4	1062	2.07	465	68
Leonhardt (1962)	190	278	5.00	30.0	28.7	1062	2.01	465	62
Leonhardt (1962)	190	278	5.00	30.0	28.7	1062	2.01	465	68
Leonhardt (1962)	190	278	6.00	30.0	28.8	1062	2.01	465	66
Leonhardt (1962)	190	274	6.00	30.0	28.8	1062	2.04	465	66
Leonhardt (1962)	100	140	3.00	15.0	29.7	226	1.62	427	21
Leonhardt (1962)	100	140	3.00	15.0	29.7	226	1.62	427	23
Leonhardt (1962)	150	210	3.00	15.0	32.1	509	1.62	413	46
Leonhardt (1962)	150	210	3.00	15.0	32.1	509	1.62	413	43
Leonhardt (1962)	150	210	3.00	15.0	32.1	509	1.62	413	43
Leonhardt (1962)	200	280	3.00	15.0	32.8	938	1.68	439	74
Leonhardt (1962)	200	280	3.00	15.0	32.8	938	1.68	439	71
Leonhardt (1962)	200	280	3.00	15.0	32.8	938	1.68	439	71
Leonhardt (1962)	100	150	3.00	30.0	36.4	199	1.33	425	22
Leonhardt (1962)	150	300	3.00	30.0	36.4	603	1.34	425	65
Leonhardt (1962)	200	450	3.00	30.0	36.4	1206	1.34	425	102
Leonhardt (1962)	225	600	3.00	30.0	36.4	1810	1.34	425	152
Leonhardt (1962)	501	142	2.46	30.0	12.0	679	0.95	427	100
Leonhardt (1962)	190	270	2.48	30.0	27.4	1062	2.07	465	82
Leonhardt (1962)	190	270	2.48	30.0	27.4	1062	2.07	465	87
Leonhardt (1962)	190	270	2.78	30.0	19.4	933	1.82	439	58
Leonhardt (1962)	190	270	2.78	30.0	19.4	911	1.78	490	75
Marti; Pralong; Thürlimann (1977)	400	162	3.95	16.0	28.1	896	1.38	542	97

Authors	b_w	d	a/d	ϕ	f'c	A_{sl}	ρ	fy	Vc
Mathey, Watstein (1963)	203	403	3.78	25.4	27.8	2083	2.54	505	88
Mathey, Watstein (1963)	203	403	3.78	25.4	23.9	2083	2.54	505	81
Mathey, Watstein (1963)	203	403	3.78	25.4	22.3	763	0.93	690	63
Mathey, Watstein (1963)	203	403	3.78	25.4	24.3	763	0.93	690	66
Mathey, Watstein (1963)	203	403	3.78	25.4	25.0	383	0.47	696	54
Mathey, Watstein (1963)	203	403	3.78	25.4	24.5	383	0.47	696	50
Mathey, Watstein (1963)	203	403	2.84	25.4	24.8	688	0.84	707	71
Mathey, Watstein (1963)	203	403	2.84	25.4	24.5	688	0.84	707	62
Mathey, Watstein (1963)	203	403	2.84	25.4	29.0	688	0.84	707	75
Moody (1954)	178	262	2.96	25.4	28.8	1007	2.17	310	60
Moody (1954)	178	267	2.90	25.4	29.5	1013	2.14	310	67
Moody (1954)	178	268	2.89	25.4	29.5	1061	2.23	310	76
Moody (1954)	178	270	2.87	25.4	29.9	1140	2.37	310	71
Moody (1954)	178	267	2.90	25.4	20.1	760	1.60	310	56
Moody (1954)	178	268	2.89	25.4	20.5	776	1.63	310	60
Moody (1954)	178	270	2.87	25.4	18.3	768	1.60	310	56
Moody (1954)	178	272	2.85	25.4	15.9	792	1.64	310	56
Moody (1954)	152	268	3.41	25.4	34.9	776	1.90	310	58
Moody (1954)	152	268	3.41	25.4	15.9	776	1.90	310	36
Moody (1954)	152	268	3.41	25.4	24.5	776	1.90	310	52
Moody (1954)	152	268	3.41	25.4	14.6	776	1.90	310	40
Moody (1954)	152	268	3.41	25.4	29.2	776	1.90	310	52
Moody (1954)	152	268	3.41	25.4	15.0	776	1.90	310	34
Moody (1954)	152	268	3.41	25.4	29.4	776	1.90	310	51
Moody (1954)	152	268	3.41	25.4	39.1	776	1.90	310	53
Moody (1954)	152	268	3.41	25.4	22.7	776	1.90	310	49
Moody (1954)	152	268	3.41	25.4	36.2	776	1.90	310	60
Moody (1954)	152	268	3.41	25.4	19.2	776	1.90	310	47
Moody (1954)	152	268	3.41	25.4	21.4	776	1.90	310	43
Moody (1954)	152	268	3.41	25.4	35.5	776	1.90	310	51
Moody (1954)	152	268	3.41	25.4	15.5	776	1.90	310	38
Morrow, Viest (1957)	305	368	4.10	25.4	14.0	2077	1.85	471	100
Morrow, Viest (1957)	305	375	4.03	25.4	23.7	2756	2.41	330	138
Morrow, Viest (1957)	305	368	4.10	25.4	25.9	2077	1.85	441	122
Morrow, Viest (1957)	305	368	4.10	25.4	27.0	1387	1.24	429	109
Morrow, Viest (1957)	308	356	4.25	25.4	37.9	4149	3.79	439	178
Morrow, Viest (1957)	305	372	4.07	25.4	43.4	2077	1.83	466	137
Morrow, Viest (1957)	305	365	5.11	25.4	15.5	2077	1.87	462	89
Morrow, Viest (1957)	305	368	5.07	25.4	25.9	2756	2.46	436	132
Morrow, Viest (1957)	305	356	5.25	25.4	42.7	4149	3.83	435	178
Morrow, Viest (1957)	305	363	6.11	25.4	25.9	2077	1.88	465	111
Morrow, Viest (1957)	305	368	3.00	25.4	33.0	2077	1.85	378	156
Mphonde, Frantz (1984)	152	298	3.49		20.2	1520	3.34	414	65
Mphonde, Frantz (1984)	152	298	3.49		26.4	1061	2.33	414	67
Mphonde, Frantz (1984)	152	298	3.49		36.7	1520	3.34	414	82
Mphonde, Frantz (1984)	152	298	3.49		40.6	1520	3.34	414	83
Mphonde, Frantz (1984)	152	298	3.49		73.0	1520	3.34	414	90
Mphonde, Frantz (1984)	152	298	3.49		72.7	1520	3.34	414	89
Mphonde, Frantz (1984)	152	298	3.49		79.3	1520	3.34	414	93
Mphonde, Frantz (1984)	152	298	3.49		91.3	1520	3.34	414	100
Mphonde, Frantz (1984)	152	298	3.49		89.5	1520	3.34	414	98
Mphonde, Frantz (1984)	152	298	2.41		20.1	1520	3.34	414	78
Mphonde, Frantz (1984)	152	298	2.41		44.0	1520	3.34	414	118
Mphonde, Frantz (1984)	152	298	2.41		77.2	1520	3.34	414	111
Mphonde, Frantz (1984)	152	298	2.41		81.6	1520	3.34	414	178
Mphonde, Frantz (1984)	152	298	2.41		67.6	1520	3.34	414	206
Podgorniak-Stanik (1998)	300	925	2.88	10.0	89.3	1400	0.50	550	164
Podgorniak-Stanik (1998)	300	925	2.88	10.0	35.2	2100	0.76	550	192
Podgorniak-Stanik (1998)	300	925	2.88	10.0	94.1	2100	0.76	550	193
Podgorniak-Stanik (1998)	300	450	2.92	10.0	35.2	1100	0.81	486	132
Podgorniak-Stanik (1998)	300	450	2.92	10.0	94.1	1100	0.81	486	132
Podgorniak-Stanik (1998)	300	225	2.95	10.0	35.2	600	0.89	437	73
Podgorniak-Stanik (1998)	300	110	2.96	10.0	35.2	300	0.91	458	40
Rajagopalan; Ferguson (1968)	152	265	4.22	13.0	22.5	697	1.73	655	40
Rajagopalan; Ferguson (1968)	154	259	3.93	13.0	34.7	568	1.43	655	36
Rajagopalan; Ferguson (1968)	154	265	3.83	13.0	31.4	400	0.98	655	37
Rajagopalan; Ferguson (1968)	152	267	4.19	13.0	27.5	329	0.81	524	31
Rajagopalan; Ferguson (1968)	152	268	4.17	13.0	31.4	258	0.63	524	28
Rajagopalan; Ferguson (1968)	152	262	4.27	13.0	26.5	211	0.53	1779	34
Rajagopalan; Ferguson (1968)	152	262	4.27	13.0	23.8	211	0.53	1779	24
Rajagopalan; Ferguson (1968)	151	267	4.18	13.0	29.5	141	0.35	1779	27
Rajagopalan; Ferguson (1968)	152	268	4.17	13.0	27.2	103	0.25	1779	30
Rajagopalan; Ferguson (1968)	153	268	4.16	13.0	28.2	103	0.25	1779	25
Reineck; Koch; Schlaich (1978)	500	226	3.50	16.0	24.5	887	0.79	501	102
Reineck; Koch; Schlaich (1978)	500	226	2.50	16.0	24.5	887	0.79	501	118
Reineck; Koch; Schlaich (1978)	500	225	2.50	16.0	23.4	1569	1.39	441	140

Authors	b_w	d	a/d	ϕ	f'c	A_{sl}	ρ	fy	Vc
Remmel (1991)	150	165	4.00	16.0	80.8	462	1.87	523	46
Remmel (1991)	150	165	3.06	16.0	80.8	462	1.87	523	48
Remmel (1991)	150	160	4.00	16.0	80.3	982	4.09	474	58
Remmel (1991)	150	160	3.06	16.0	80.3	982	4.09	474	60
Ruesch, Haugli (1962)	90	111	3.60	30.0	21.9	265	2.65	481	15
Ruesch, Haugli (1962)	120	199	3.60	30.0	21.9	634	2.65	407	30
Ruesch, Haugli (1962)	180	262	3.62	30.0	23.0	1246	2.64	412	55
Scholz (1994)	200	372	3.00	16.0	76.6	603	0.81	500	83
Scholz (1994)	200	362	3.00	16.0	92.0	1407	1.94	500	121
Scholz (1994)	200	362	4.00	16.0	92.0	1407	1.94	500	121
Taylor (1968)	203	370	3.02	9.5	27.4	776	1.03	350	62
Taylor (1968)	203	370	3.02	9.5	31.6	1164	1.55	350	92
Taylor (1968)	203	370	3.02	9.5	27.4	776	1.03	350	76
Taylor (1968)	203	370	3.02	9.5	31.6	1164	1.55	350	101
Taylor (1968)	203	370	3.02	9.5	30.0	776	1.03	350	76
Taylor (1968)	203	370	2.47	9.5	28.4	776	1.03	350	81
Taylor (1968)	203	370	2.47	9.5	28.4	776	1.03	350	81
Taylor (1972)	200	465	3.00	38.0	25.5	1257	1.35	420	104
Taylor (1972)	200	465	3.00	19.0	20.9	1257	1.35	420	87
Taylor (1972)	200	465	3.00	9.0	27.0	1257	1.35	420	85
Taylor (1972)	400	930	3.00	38.0	27.3	5027	1.35	420	358
Taylor (1972)	400	930	3.00	19.0	21.5	5027	1.35	420	328
Thorenfeldt, Drangshold (1990)	150	221	3.00	16.0	51.3	603	1.82	500	58
Thorenfeldt, Drangshold (1990)	150	207	4.00	16.0	51.3	1005	3.24	500	70
Thorenfeldt, Drangshold (1990)	150	207	3.00	16.0	51.3	1005	3.24	500	83
Thorenfeldt, Drangshold (1990)	150	221	3.00	16.0	73.9	603	1.82	500	68
Thorenfeldt, Drangshold (1990)	150	207	4.00	16.0	73.9	1005	3.24	500	78
Thorenfeldt, Drangshold (1990)	150	207	3.00	16.0	73.9	1005	3.24	500	83
Thorenfeldt, Drangshold (1990)	150	207	4.00	16.0	55.1	1005	3.24	500	68
Thorenfeldt, Drangshold (1990)	150	207	3.00	16.0	55.1	1005	3.24	500	83
Thorenfeldt, Drangshold (1990)	150	207	4.00	16.0	82.1	1005	3.24	500	86
Thorenfeldt, Drangshold (1990)	150	207	3.00	16.0	82.1	1005	3.24	500	107
Thorenfeldt, Drangshold (1990)	150	221	3.00	16.0	92.8	603	1.82	500	56
Thorenfeldt, Drangshold (1990)	150	207	4.00	16.0	92.8	1005	3.24	500	77
Thorenfeldt, Drangshold (1990)	150	207	3.00	16.0	92.8	1005	3.24	500	78
Thorenfeldt, Drangshold (1990)	300	442	3.00	16.0	73.9	2413	1.82	500	180
Thorenfeldt, Drangshold (1990)	300	414	3.00	16.0	73.9	4021	3.24	500	281
Walraven (1978)	200	420	3.00		22.9	622	0.74	440	71
Walraven (1978)	200	720	3.00		23.2	1140	0.79	440	101
Xie, Ahmad, Yu (1994)	127	216	3.00	19.1	36.5	568	2.07	421	37
Xie, Ahmad, Yu (1994)	127	216	3.00	19.1	95.9	568	2.07	421	46
Yoon, Cook, Mitchell (1996)	375	655	3.23	20.0	34.2	7000	2.85	400	249
Yoon, Cook, Mitchell (1996)	375	655	3.23	10.0	63.7	7000	2.85	400	296
Yoon, Cook, Mitchell (1996)	375	655	3.23	10.0	82.7	7000	2.85	400	327

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