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Filled Function with Single Parameter Application on Minimum Pumping Power Air Flow Over Staggered Adiabatic Flat Tubes

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Keywords:
Filled function; Single parameter; Constructal design; Minimum pumping power; Staggered arrangement.

Highlights:

- *A new engineering application of a filled function with a single parameter.*
- *The optimize a minimum of pumping in flow over two row staggered flat tubes.*
- *In the investigated in current study based on the constructal design method.*

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Abstract: An engineering application of a filled function with a single parameter that has been optimized for a minimum of pumping in flow over two row staggered flat tubes, the minimum of pumping power in air flow are investigated in current study based on the constructal design method. Nine values of the Reynolds number between 5 to 300 and three values of the longitudinal pitch are 1, 2, and 3. The tubes were spaced apart in dimensionless tube-to-tube spacing from 0.75 to 2.5. Pumping power can be minimum with respect to tube spacing, according to numerical optimization results for flat tubes arrangements. In longitudinal pitch 1.0, the minimum of pumping power (optimal value) with transverse pitch $(\phi_1/2d)$ for each Reynolds number as following: Re, $\phi_1/2d$; 5 at 1.3, 15 at 1.2, 50 at 1.3, 75 at 1.25, 100 at 1.5, 150 at 1.25, 200 at 1.2, 250 at 1 and 300 at 1, receptively.

الدالة الممتلئة ذات المعامل الواحد وتطبيقها على الحد الأدنى من قوة الضخ لتدفق الهواء فوق الأنابيب المسطحة المتداخلة الاديباتية

محمد إبراهيم احمد تحسين [١]، قاسم ناصر حسين [١]، شهاب احمد ابراهيم [٢]، تحسين احمد تحسين [٣]، محاذير اسحاق [٤]، محمد مستفيض الرحمن [٥]

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الخلاصة

إن طبيعة التطبيق الهندسي للدالة الممتلئة ذات معامل واحد تم تحسينه لإيجاد الحد الأدنى من قوة الضخ للتدفق عبر صفين من الأنابيب المسطحة المتداخلة، تم التحقق من الحد الأدنى لقوة الضخ لتدفق الهواء في الدراسة الحالية بناءً على طريقة تصميم التشييد (constructal design). استخدم تسع قيم لعدد رينولدز بين ٥ إلى ٣٠٠ وثلاث قيم للخطوة الطولية وهي ١ و ٢ و ٣. اما الخطوة العمودية وهي المسافة بين أنبوب إلى أنبوب من ٠,٧٥ إلى ٢,٥. يمكن أن تكون قوة الضخ في حدها الأدنى فيما يتعلق بالخطوة العمودية للأنابيب، وفقًا لنتائج التحسين العددي لترتيبات الأنابيب المسطحة. عند الخطوة الطولية ١,٠، يكون الحد الأدنى لقوة الضخ (القيمة المثلى) مع الخطوة العمودية ($\phi_1/2d$) لكل عدد رينولدز على النحو التالي: Re ، $\phi_1/2d$ ؛ ٥ عند ١,٣، و١٥ عند ١,٢، و٥٠ عند ١,٣، و ٧٥ عند ١,٢٥، و ١٠٠ عند ١,٥، و ١٥٠ عند ١,٢٥، و ٢٠٠ عند ١,٢، و ٢٥٠ عند ١، و ٣٠٠ عند ١، على التوالي.

الكلمات الدالة: الدالة الممتلئة، معامل واحد، تصميم التشيد، القيمة الدنيا لقوة الضخ، الترتيب المتداخل.

1.INTRODUCTION

Several domains, including engineering, finance, management, decision science, and others, have extensive use for global optimization. Finding a solution with the least or greatest objective function value is the goal of global optimization. In this article, we mostly cover how to get the objective function's global minimum. There are a variety of local optimization techniques accessible for issues with a single minimum. To get out of the existing local minimum and locate a superior one is the main challenge for global optimization. The filled function method, first forth by Ge [1] and Lin et al. [2], is among the most effective ways to address this problem. Substantial research effort has been exerted to improve the efficiency of heat exchangers because of the widespread use of these devices in industrial, transportation, and domestic applications, including thermal power plants, means of transport, heating and air conditioning systems, electronic equipment, and space vehicles [3]. Increasing the efficiency of heat exchangers would greatly reduce the cost, space, and materials required in their use [4]. Several investigations have focused on the study of fluid flow and heat transfer around cylinder-shaped [5-8]. The describe constructs design of different-shaped cylindrical cooled by natural convection [9,10], by forced convection [11,12]. Forced convection from a bank of flat tubes with staggered and in-line arrangements is presented in Refs [13-16] and circular tube [17]. In order to, maximize heat transfer in circular and elliptical tube geometries under a Newtonian fluid cross-flow, Matos et al. [18] carried out a numerical and experimental investigation. According to experimental findings, an elliptical design transfers heat up to 20% better than a circular one, the turbulent forced convection [19]. Abbas et al. [20] the configuration of longitudinally finned tubes cooled by forced convection is determined using the constructal design method. The two degrees of freedom are the tube-to-tube spacing and longitudinal fins length are presented in this study. A three-fin location inside the domain is investigated. The results showed that the spacing between the un-finned and finned tubes may be changed to an ideal spacing for the Bejan numbers and fin location taken into consideration, resulting in the maximum heat transfer from the tubes to the coolant. The constructal design method to investigate how viscoplasticity influences system performance and what geometric designs are ideal dependent on fluid rheology in a row of circular tubes tested with cross-flows of viscoplastic Bingham fluids are presented by Severo et al. [21]. The performance indicator for the constructal design method was the heat transfer density and the degree of freedom was tube-to-tube space. The Bejan number between 10^4-10^5 and Bigham number are 1, 10 and 100 were tested. The result show that, If Bejan number increases the dimensionless heat transfer density increases and increases of Bingham number dimensionless heat transfer density decreases. The goal of the current work is to CFD investigate and illustrate an engineering application of a filled function with a single parameter that has been optimized for a minimum of pumping in flow over staggered flat tubes. The Reynolds number in this study is calculated using the small flat tube diameter and inlet velocity. The different of transverse and longitudinal pitches are investigated. The transverse pitch between 0.75 to 2.5, a longitudinal pitch is 1, 2, and 2.5 with the Reynolds number range 5 to 300.

2.THEORY

2.1.Characteristics of a New Filled Function Method

To solve the Eq. (1), we provide the following new filled function at a local minimum x_k^* with single parameter:

$$\boldsymbol{F(x, x_k^*, \sigma) = \frac{1}{\|x - x_k^*\|^2 + \sigma} z(f(x) - f(x_k^*), \sigma)}, \qquad \textbf{(1)}$$

therefore

$$\boldsymbol{z(t, \sigma) = \begin{cases} 1, & t \geq 0, \\ 2 - e^{-\left(\frac{t}{\sigma}\right)}, & t < 0, \end{cases}}$$

Likewise, $0 < \sigma \leq 1$ is parameter.

This filled function's novel idea is to place a lot of stationary points in the lower basin, $\Omega_2 = \{x | f(x) < f(x_k^*), x \in \Omega\}$. The filled function just needs to find any stationary point in Ω_2 that may be utilized as an initial for minimizing the objective function to produce a lower minimum; it does not actually need to descend into the lower basin. The aforementioned concept offers various benefits, including the ability to cut down on time and evaluation-both of which are crucial in this situation. Also, the parameter σ is used to either increase or decrease the number of stationary points in the interval Ω_2, so we must carefully select because if it is too small, we risk losing some of the lower minimums where the value of the function is similar to the value at the first minimum (see Fig. 1). The hat is controlled by the simple to adjust parameter $0 < \sigma \leq 1$ in the phrase $\frac{1}{\sigma + \|x - x_k^*\|^2}$. The function $F(x, x_k^*, \sigma)$ is a filled function according to Eq. (1), as shown by the following theorems.

2.2.Parameters of Interest

Figure 2 (a) shows a conventional two-row tube heat exchanger with a generally staggered arrangement. The heat exchanger envelope $W_x W_y$, which is retained within limits, is the first consideration in the engineering design problem. Furthermore, the pumping power and the overall pressure drop are minimized. The following definitions will be used [22]:

$$\boldsymbol{\dot{H}P = \frac{\dot{m}}{\rho} \Delta p} \qquad \textbf{(2)}$$

where $\dot{m}$ is the inlet overall mass flow rate (kg/s).

As indicated in Fig. 2(a), an elemental channel is defined as the total of all unit cells in direction z. The resultant number of elemental channels

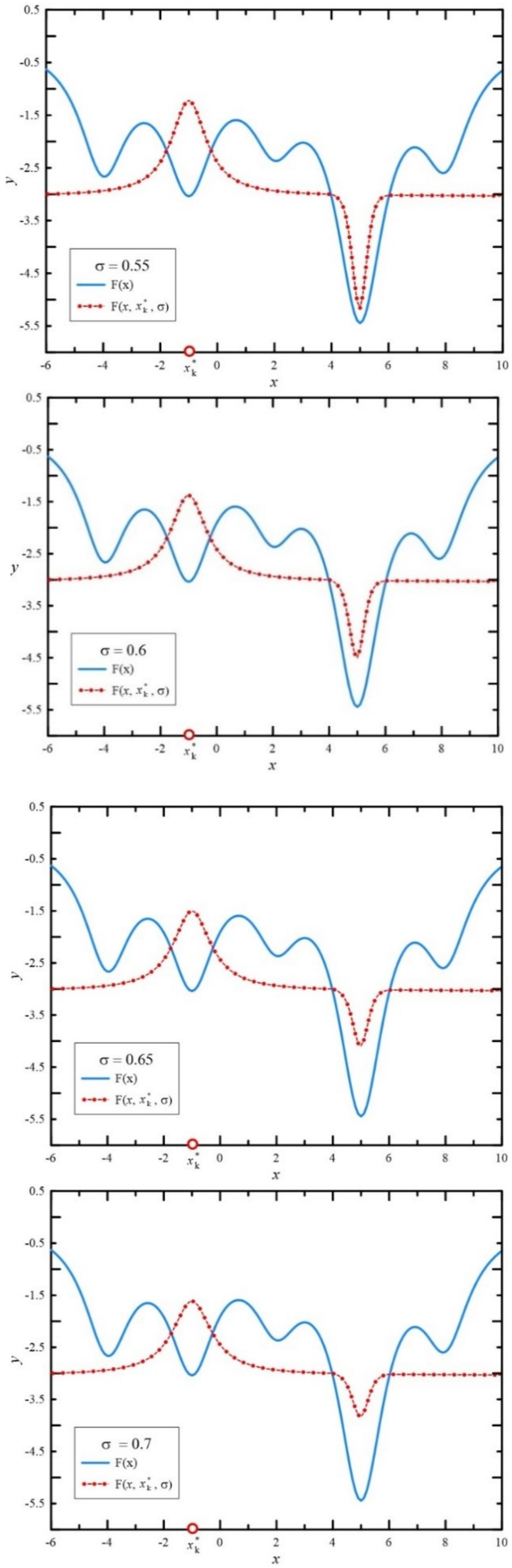


Fig. 1 Several Various Parameter Values σ and How they Affect the Function $F(x, x_k^*)$.

in the heat exchanger, the overall mass flow rate entering the heat exchanger can be written as:

$$\dot{m} = \left(\frac{\phi_1}{d} + 1\right)\rho u_\infty dL \tag{3}$$

Thus, the objective function (dimensionless pumping power) is expressed as follows [22]:

$$\widetilde{H}P = \frac{\dot{H}P}{\rho u_\infty^3 dL} = \left(\frac{\phi_1}{d} + 1\right)\Delta p^* \tag{4}$$

in the expression: $\Delta p^* = \Delta p/(\rho u_\infty^2)$, is the pressure drop dimensionless.

2.3.Problem Description

For the conservation of mass, momentum, and energy in two dimensions, equations are developed. With the assumption that the flow is two-dimensionally stable and laminar and that the fluid is incompressible with constant thermal properties, velocity uses the Cartesian components u^* and v^*. According to Bejan and Kraus [4], the four equations for the dependent variables u^*, v^*, p^* and T^* form the governing equations:

Continuity equation:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{5}$$

Momentum (Navier-Stokes) equation:

X-direction (u momentum):

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{\partial P^*}{\partial x^*} + \frac{1}{Re_d}\left[\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right] \tag{6}$$

Y-direction (v momentum):

$$u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} = -\frac{\partial P^*}{\partial y^*} + \frac{1}{Re_d}\left[\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right] \tag{7}$$

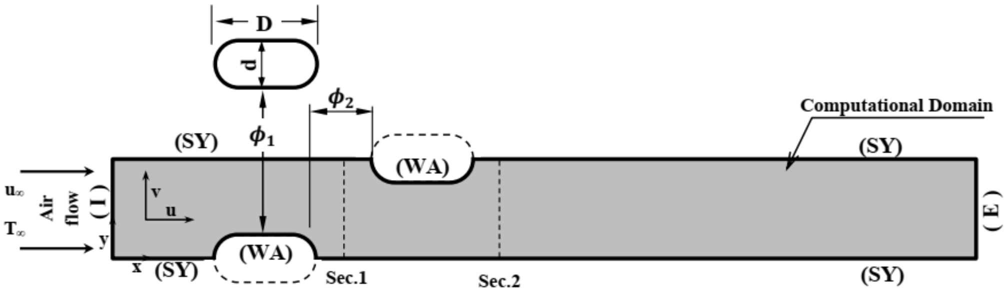
Energy equation:

$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{Pr\,Re_d}\left[\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right] \tag{8}$$

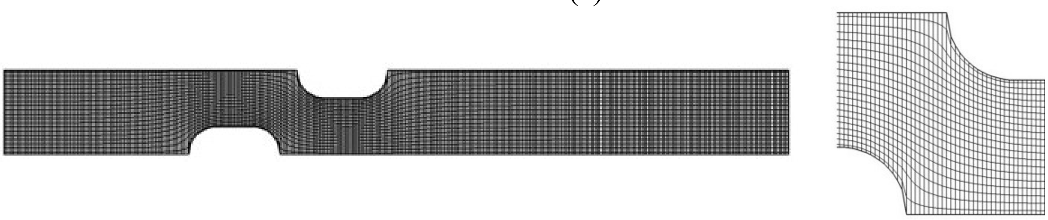
The governing equations were transformed into dimensionless form upon incorporating the following non-dimensional variables:

$$(x^*, y^*) = \frac{(x, y)}{d},\quad (u^*, v^*) = \frac{(u, v)}{u_\infty},$$
$$\Delta p^* = \frac{\Delta p}{\rho(u_\infty)^2},\ T^* = \frac{T - T_\infty}{T_w - T_\infty},$$
$$Re_d = \frac{u_\infty d}{\nu},\quad Pr = \frac{C_p\,\mu}{k}$$

The physical system taken into consideration for this investigation is shown in Fig. 2(b). Uniform inlet velocity, fully developed outflow, and a combination of symmetry and no-slip tube surfaces on the bottom and top boundaries define the boundary conditions for the solution domain. Table 1 displays the boundary conditions that are required for u^*, v^* and T^*. Using FORTRAN 90, the governing equations are numerically solved. The finite-volume method was used to discretize the continuity, momentum, and energy equations, and the computer programmer was able to solve them. The method relies on a non-orthogonal coordinate system with Cartesian velocity components, a non-staggered (collocated) grid [23], and the SIMPLE algorithm [24]. For more details of the gird independence test and model validation can see the Refs. [25,26].



(a)



(b)

Fig. 2 Schematic Representation of the Problem: (a) Computational Domain and Boundary Conditions, (b) Computation Grids and Mesh Detail Near the Wall of Tube.

Table 1 The Boundary Conditions.

Location	u^*	v^*	T^*
Inlet (I)	1	0	0
Exit (E)	$\frac{\partial u^*}{\partial x^*}=0$	$\frac{\partial v^*}{\partial x^*} = 0$	$\frac{\partial T^*}{\partial x^*} = 0$
Tube wall (WA)	0	0	1
Symmetry (SY)	$\frac{\partial u^*}{\partial y^*} = 0$	0	$\frac{\partial T^*}{\partial y^*} = 0$

3.RESULTS AND DISCUSSION

These case studies and parameter combinations are summarized in Fig. 3. For nine values of the Reynolds number (5 to 300), there were three values of the longitudinal pitch: 1, 2, and 3, assuming a fixed Prandtl number of 0.71 (air flow). The tubes were spaced apart in dimensionless steps ranging from 0.75 to 2.5. Thirty-three simulations were run for each of the Reynolds number case studies in order to establish a foundation for the study with sufficient accuracy regarding the ideal step size and pumping power. A set of 297 simulations were performed as a result. An Intel Core i7 computer with 12 GB of RAM was used to solve the simulations. Each simulation took an average of 26 minutes to complete, using 129 hours of computation time.

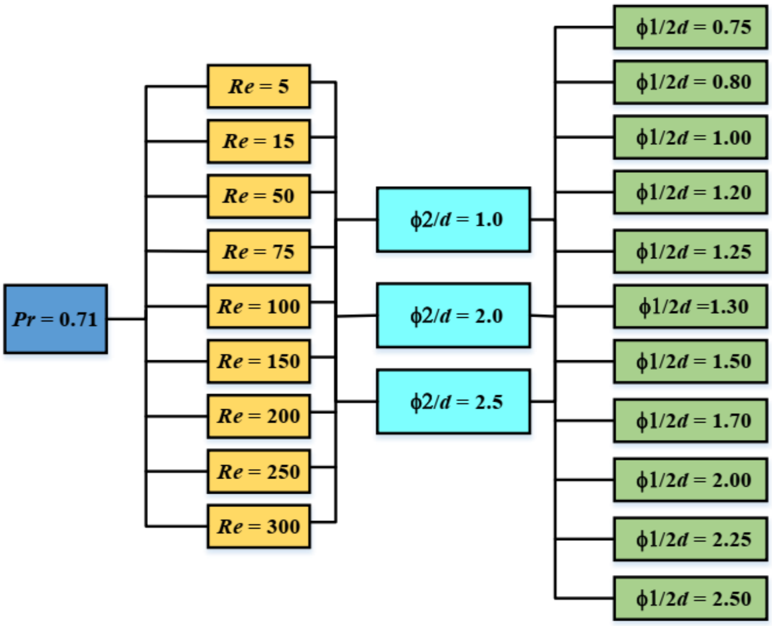


Fig. 3 Case Studies and Parameter Combinations from the Current Study are Summarized.

In order to find the flat tube heat exchanger design's geometrical optimum for a minimum value of dimensionless pumping power, numerical optimization was carried out. According to the following, the optimization process was carried out for each tested Reynolds number:

(a) for a fixed tube diameter (small and big), using Eq. (4) to calculated the pumping power for the range of $0.75 \leq \phi_1/2d \leq 2.5$,

(b) the same procedure was repeated for different ϕ_2/d. Fig. 4 presented the results of dimensionless pumping power minimum for flat tube configurations in with respect $\phi_1/2d$ for different Reynolds number at $\phi_2/d = 1.0$. From the figure, it can be clearly seen that the minimum pumping power is observed in all cases of Reynolds number. Note also that the minimum value of pumping power (optimal value) with $\phi_1/2d$ pitch for each Reynolds number as following: Re, $\phi_1/2d$; 5 at 1.3, 15 at 1.2, 50 at 1.3, 75 at 1.25, 100 at 1.5, 150 at 1.25, 200 at 1.2, 250 at 1 and 300 at 1, receptively. For illustrative purposes, the pumping power increases with Reynolds number decreases for all cases tested. This result was expected based on the theoretical analysis that was previously

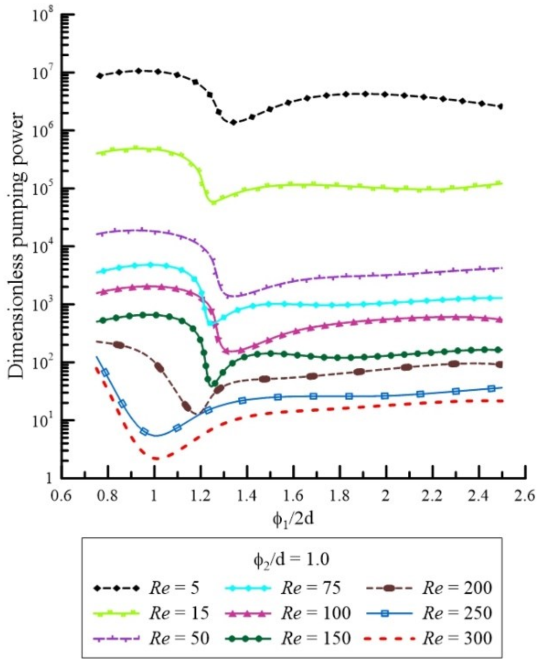


Fig. 4 Results of Dimensionless Pumping Power Minimum for Flat Tube Configurations in with Respect to Tube-to-Tube Spacing for Different Reynolds Number at $\phi_2/d = 1.0$.

discussed in the text, due to the higher pressure drop with low Reynolds number according the relationship $\Delta p/(\rho u_\infty^2)$. The work indicates that a configuration performs better at lower pitch ratios and approach velocities, but performs worse at greater approach velocities and widely spaced tube banks. The results of the dimensionless pumping power with respect tube-to-tube distance for several Reynolds number at $\phi_2/d = 2.0$ are reported in Fig. 5. It can be found that, the minimum pumping power (optimal value) with Reynolds number are $Re = 5$ at $\phi_1/2d = 1.2$, $Re = 15$ at $\phi_1/2d = 1.3$, Re = 50 at $\phi_1/2d = 1.3$, Re = 75 at $\phi_1/2d = 1.5$, $Re = 100$ at $\phi_1/2d = 1.2$, $Re = 150$ at $\phi_1/2d = 1.25$, $Re = 200$ at $\phi_1/2d = 1.25$, $Re = 250$ at $\phi_1/2d = 1.25$, and $Re = 300$ $\phi_1/2d = 1.2$.

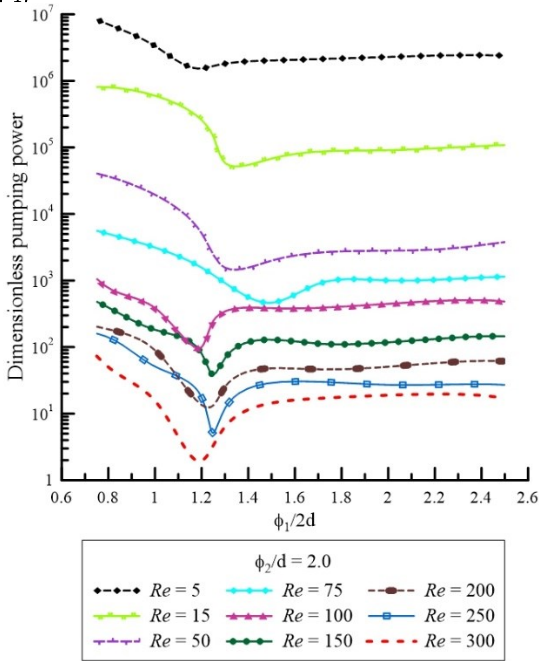


Fig. 5 Results of Dimensionless Pumping Power Minimum for Flat Tube Configurations in with Respect to Tube-To-Tube Spacing for Different Reynolds Number at $\phi_2/d = 2.0$.

For other longitudinal pitch $\phi_2/d = 2.5$, the dimensionless pumping power versus the tube-to- tube spacing with several Reynolds number are shown in Fig. 6. It is apparent from figure that the minimum pumping power (optimal value) with $\phi_1/2d$ pitch for each Reynolds number as following: Re, $\phi_1/2d$; 5 at 1, 15 at 1.25, 50 at 1.25, 75 at 1.3, 100 at 1.2, 150 at 1.2, 200 at 1.25, 250 at 1.25 and 300 at 1.2, receptively. The effect of Reynolds number on dimensionless pumping power minimum results for flat tube array with respect to tube -to-tube spacing at $\phi_2/d = 1.0$ as is illustrated in Fig. 7. From the figure, it can be clearly seen that with the increase of Reynolds number the pumping power decrease for all study cases. It can be observed that the lowest value of pumping power occurs at $\phi_1/2d = 2.5$ for each Reynolds number and highest value at $\phi_1/2d = 0.75$. For fixed length of domain and according on this result, it is suggested to install a large of $\phi_1/2d$ pitch with several Reynolds number to get the minimum pumping power values. Figs. 8 and 9 provide the same information for a longitudinal pitch $\phi_2/d = 2.0$, and $\phi_2/d = 2.5$, respectively.

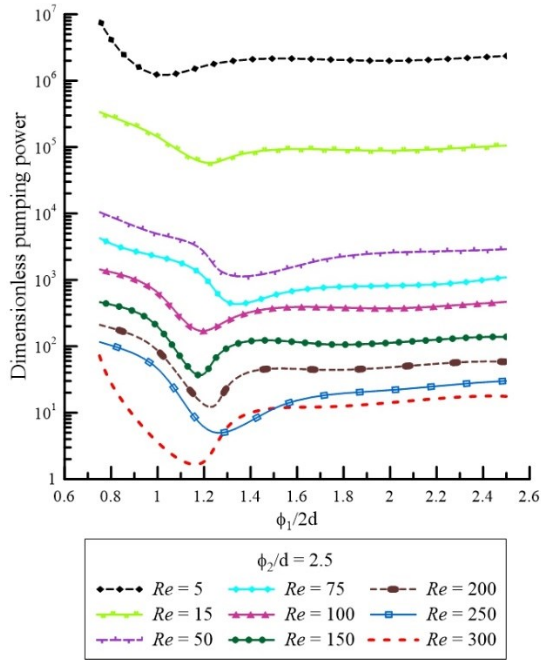


Fig. 6 Results of Dimensionless Pumping Power Minimum for Flat Tube Configurations in with Respect to Tube-to-Tube Spacing for Different Reynolds Number at $\phi_2/d = 2.5$.

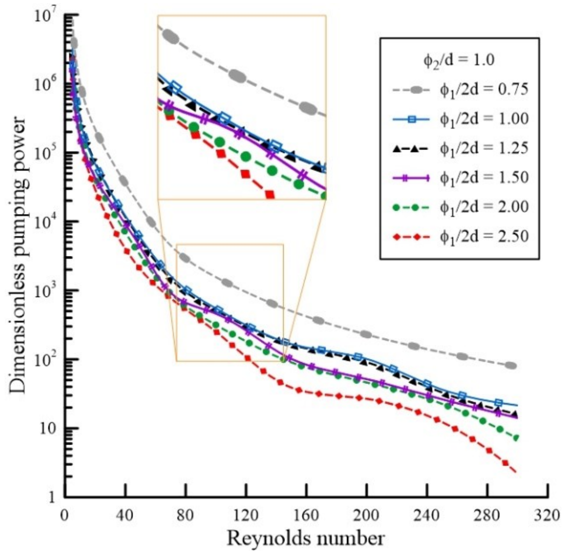


Fig. 7 The Influence of Reynolds number on Dimensionless Pumping Power Minimum Results for Flat Tube Array with Respect to Tube -to-Tube Spacing at $\phi_2/d = 1.0$.

At the optimum tube-to-tube spacing, the effects of the Reynolds number on the streamlines are investigated. The streamlines over the tubes for different Reynolds number at $\phi_1/2d = 1.5$ and at $\phi_2/d = 1.0$ are presented in Fig. 10. Due to the significantly bigger recirculation (vortices), this figure shows the domain in the second tube. In comparison to the tube after it in the flow direction, the recirculation behind the first tube is less in size.

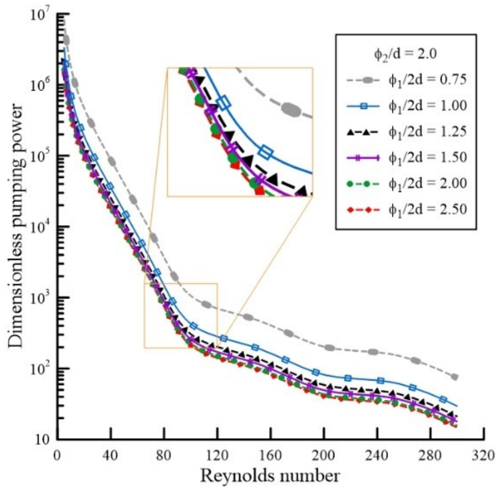


Fig. 8 The Influence of Reynolds Number on Dimensionless Pumping Power Minimum Results for Flat Tube Array with Respect to Tube -to-Tube Spacing at $\phi_2/d = 2.0$.

Behind the first tube is a small recirculation zone, and as the Reynolds number increases, increasing the size of the recirculation zone. For illustrative purposes, for the lower Reynolds number (i.e., < 50) the recirculation zone not appears. Behind each of the tubes in this instance, close to the inlet, is a very small flow recirculation zone. These vortices cause instability in the field of flow through concave curvature destabilization of the boundary layer [23, 24]; thus, it is expected to be shed in circulation. The streamlines over the tubes for different $\phi_1/2d$ at $Re = 200$ and at $\phi_2/d = 2.5$ has been demonstrated from Fig. 11. It can be seen the recirculation zone decreases with increase the $\phi_1/2d$ because the channel between two tubes is increase.

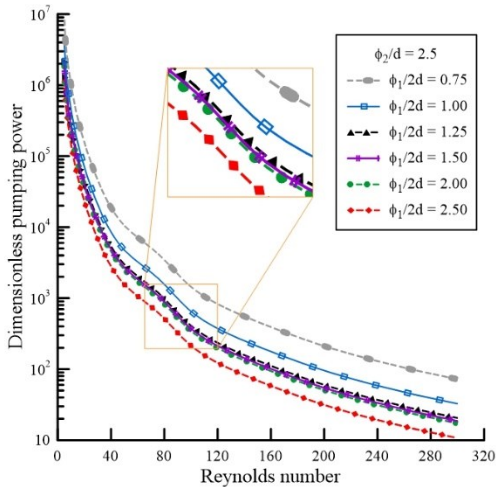


Fig. 9 The Influence of Reynolds Number on Dimensionless Pumping Power Minimum Results for Flat Tube Array with Respect to Tube -to-Tube Spacing at $\phi_2/d = 2.5$.

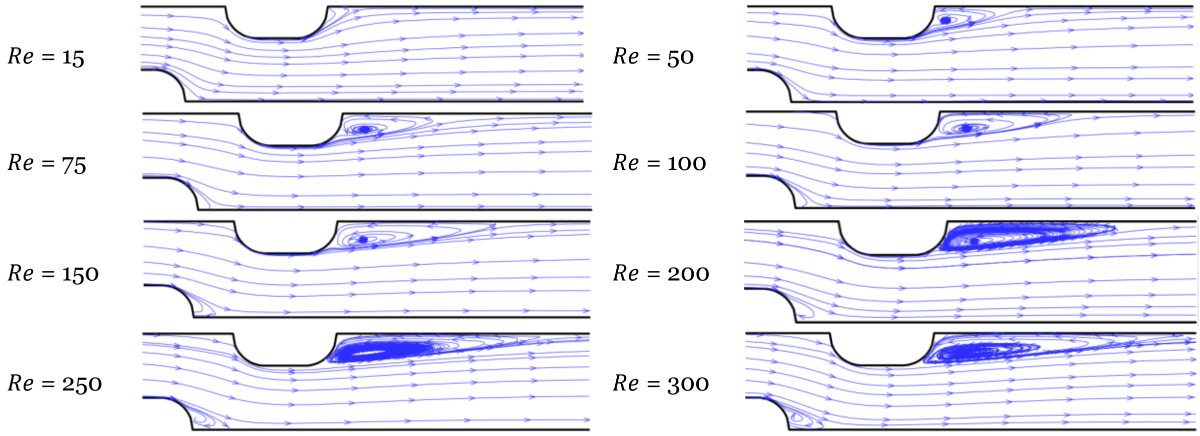


Fig. 10 The Streamlines Pattern Over the Tubes for Different Reynolds Number at $\phi_1/2d = 1.5$ and at $\phi_2/d = 1.0$.

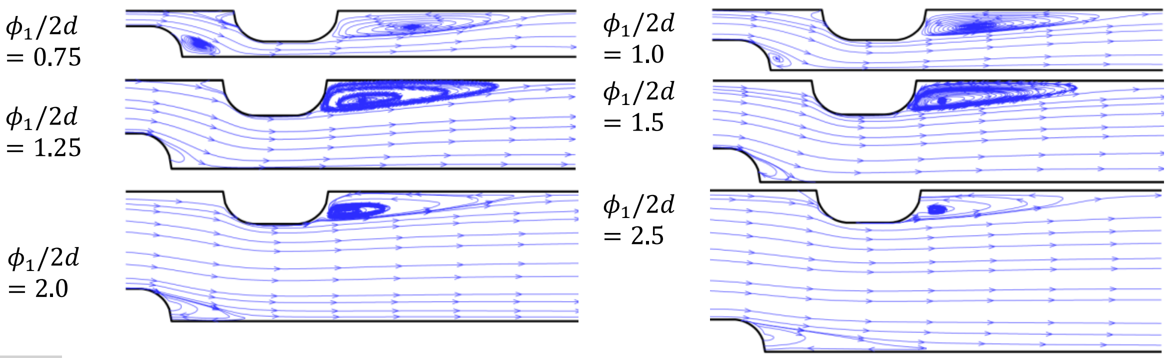


Fig. 11 The Streamlines Pattern Over the Tubes for Different $\phi_1/2d$ at $Re = 200$ and at $\phi_2/d = 2.5$.

4.CONCLUSIONS

This study is engineering application of filled function with single parameter for air flow over tubes. The work presents a numerical optimization investigation for two rows of flat tubes in a staggered arrangement to show that the optimal configurations can be identified so that the minimal pumping power appears for a given fixed volume. Several CFD samples were tested in a channel flow under laminar forced convection. This work carries out a constructal design from flat tubes in a crossflow with three degrees of freedom.

The following is a summary of the current study key conclusions.

- It was found the minimum value of pumping power is one application of filled function with single parameter with several ϕ_2/d and Re number.
- Increase of Reynolds number the pumping power decrease for all study cases.
- The minimum value of pumping power (optimal value) with $\phi_1/2d$ pitch for each Reynolds number as following: Re, $\phi_1/2d$; 5 at 1.3, 15 at 1.2, 50 at 1.3, 75 at 1.25, 100 at 1.5, 150 at 1.25, 200 at 1.2, 250 at 1 and 300 at 1, receptively.
- For future work, the present technique can be extended for many engineering problems, such that the design of any flow system in order to find the optimal value of pumping power (minimum point).

NOMENCLATURE

Cp	specific heat, J/(kg. K)
d	transverse tube diameter, m
D	longitudinal tube diameter, m
$\dot{H}P$	pumping power, W
$\widetilde{H}P$	dimensionless pumping power
L	tube length (m)
k	thermal conductivity, W/(m. K)
$\dot{m}$	mass flow rate of air entering an elemental channel, kg/s
Pr	Prandtl number
Re	Reynolds number
T	temperature, ℃
T^*	dimensionless temperature
u	x-direction velocity, m/s
u^*	dimensionless x-direction velocity
v	y-direction velocity, m/s
v^*	dimensionless y-direction velocity
x	variable
x_k^*	local minimum
Greek Symbols	
ϕ	tube-to-tube spacing
Δp	pressure drop, Pa
Δp^*	pressure drop dimensionless
ρ	density, kg/m^3
υ	kinematic viscosity, m^2/s
μ	dynamic viscosity, kg/(m·s)
σ	parameter value in Eq. (1)
Ω_2	domain interval
Subscripts	
w	tube surface
∞	free stream

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